Model structures and automata

András Kornai

Algebra Department, Budapest University of Technology and Economics
and
SZTAKI Institute of Computer Science

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Two magisteria: logic and machine learning

	logic	**ml**
typically	discrete	continuous
math	algebra-like	calculus-like
stats	irrelevant	indispensible
types	load-bearing	vestigal
subdomains	liked	hated

- Early effort at synthesis (Markov Logic) Richardson and Domingos, 2006; Domingos and Lowd, 2009 did not really have a strong logic flavor (graphical models still don't)
- Current efforts (Vector Semantics, Tensor Logic) Kornai, 2023; Domingos, 2025 may fare better
- Plan of the talk: describe current state of overlap and suggest directions for better hybrid systems

Syntax of logic

Rough fit with formal language theory:

- Two infinite vocabularies: *variables* and *(extralogical) constants*
- Terminal vocabulary: parens, quantifiers, Booleans, ...
- Terms, formulas
- 3SAT, FOL, HOL
- Syntax (whether a formula or a proof is well-formed) fits with context-sensitive ($\mathcal{L}_1$) family
- Natural language fits in *mildly* context-sensitive ($\mathcal{L}_{1.5}$) family.
- Conjecture (Marsh and Partee, 1984) logical syntax fits there too
- In vector semantics, only two variables (grammatical 'agent' and 'patient') are used, and only a finite number of constants (ballpark $10^4 - 10^6$) are required. Tarski and Givant, 1987 requires three!
- With these limitations, no infinite vocabularies, and context-free logical syntax is sufficient

Semantics

Tarski-style semantics

- Two-valued logic throughout
- Reist theory: there are worlds composed of objects and relations
- Standard (Kripke-enhanced) story: satisfaction in (nearby) models
- Ontology: any usual variant of set theory is fine
- Here we will make use of 'clouds' and 'clocks' Popper, 1966
- Clouds are *point clouds*, finite samples from possibly infinite distributions in (multi)linear space. They give us the best handle on extralogical constants. Instead of a few 'logical' constants (Boolean operators, term-binding operators) we want to extend 'logical' to things like *vertical* or *choose*. VS claims such notions are also subject to logical analysis
- Clocks will be model structures endowed with automata-theoretic *transitions* and possibly *input/output*

What we want

- Turn certain key notions from extralogical to logic-internal
- One of these notions in *live, life, living*
- We want model structures where things (automata) can live
- Typical automaton: think of ordinary computers
- Typical model structure: think of a region of space where atoms are arranged to form what we recognize as a server room
- In one second, the server room doesn't change much, but the computer performed several billion operations. We need timescales
- Birds-eye view of temporal logic
- Concentrating on the linguistically informed part beginning with the **Partee Paradox** *The temperature is 90 °F and rising*

Standard solution

- Richard Montague: *temperature* is intensional (can take different values in different possible worlds)
- Montague Grammar: models are indexed by $\langle s, t \rangle$ pairs (Montague, 1973; Gallin, 1975)
- The use of such indexes is abstract: no structure, such as $\mathbb{R}$ or $\mathbb{Z}$, is imposed on the temporal index
- Explicit temporal order begins with (Partee, 1976; Partee, 1984)
- (Kamp, 1979; Muskens, 1995) all sorts of linear orders are permitted as long as precedence is compatible with containment
- The current system also permits *frequentatives* like H. *-gat/get* as in *iszogat* 'drinks (alcohol) regularly' *igérget* 'regularly makes promises' etc.
- Simplest periodic automaton C_2 (example: *day/night*), can build more complex ones (Kornai, 2015)

Taste of vector semantics

- We can consider individual samples (occurrences in text) of words like *vertical* or *choose*
- Characterize the point clouds either by their center (works well if the cluster is very tight) or by a bounding polytope
- The polytopes are related to one another
- *vertical* 'direction, has top, has middle, has bottom, Earth pull in direction' (Has immense survival value/hardwired detector)
- *choose* '=agt choose {=pat for_ =agt}'
- Such definitions are circular, each provides an equation in a small (4-500 unknows) system of equations.
- Less than 20 true primitives, insufficient for hanging all definitions off of these
- To get to the vectors we solve the system of equations. not as good as learning them

Taste of tensor semantics

- Instead of formulas, the basic units of the formal theory are tensors of arbitrary shape $d_1, d_2, \dots, d_k$
- These are used both to model specific entities *and* to select particular 'indexable' subsets of model structures
- Existential cut: *The boy ate the sandwich* is meaningful only in worlds where there are boys and sandwiches. Ancient Greece is out.
- A neighborhood of worlds selected around the 'here and now' point in index space (actually, it's not just a single point)
- Relation composition is done by tensor multiplication
- Domingos, 2025 surfaces relations 'predicate invention' by thresholding
- Whether this is the right way remains to be seen, but the ability to devise new predicates is key to human performance

Automata

- We use Moore (emitting on state arrival) rather than Mealy (emitting on state transition)
- Deterministic or nondeterministic, full or partial flavors
- We include semiautomata (transition systems, labeled or unlabeled)
- Terminology across mechanistic devices (semiautomata, automata, transducers, machines) intentionally left vague
- Both biologically inspired and artificial neural nets are in scope
- Weighted generalizations such as probabilistic transitions would be possible but will not be discussed here
- Automata are models, models are automata

Labels/symbols

- Come from finite sets (alphabets): Σ input alphabet, O output alphabet
- Connection strengths are quantized (fp8 is extremely fine-grained)
- Proto-arithmetic on 8 bits already requires work
- System not well suited for bignum arithmetic/arbitrary precision
- Binary alphabet 0/1 created by God, the rest are the work of Man
- We begin with with Clustered Moore Automata Kornai, 2025
- Under realistic assumptions, these cannot be built up to full Turing Machines (contra Domingos 2025 and Siegelmann, 1999, in agreement with Weiss, Goldberg, and Yahav, 2018)

What automata buy us

- A simple but powerful modeling technique
- Can do effective calculations (proof theory)
- Models are simulation models, with obvious predictive power
- NOT bisimulation models! Two model structures capable of bisimulating one another are considered equivalent, one is a perfect model of the other
- Good (universal) algebraic foundations
- Built in temporality with attendant causality/mechanistic interpretability Nanda et al., 2023
- Strong links to linguistics via formal lg theory
- Possible links to biology via Free Energy Principle (Friston, 2010)

Clustered Moore Automata def.

- Like standard Moore automata $\mathcal{A} = (Q, \Sigma, O, \delta, \lambda, q_0)$ where Q is a finite set of states, Σ is the input alphabet, O is the output alphabet, $\delta : Q \times \Sigma \to Q$ is the transition function, $\lambda : Q \to O$ is the output function, and $q_0 \in Q$ is the initial state
- A CMA state can contain a smaller CMA running on a different (faster) timescale
- There are constraints on number of states, alphabet sizes, in- and out-degrees (see Kornai, 2025)
- There are only finitely many (few) timescales between `max` and `min`
- Can be used to build epistemically limited *slow* Turing Machines Kornai, 2026
- sTMs grow tetrationally with embedding depth $d =$`max-min`

Model structures are CMA

- Sets (point clouds) of model structures can be considered CMA states
- Example: models that have ‘weather’ that can be characterised by a numerical predicate `temperature` and are connected by a shared temporal order. The Partee Paradox can be resolved trivially *as long as there is only one model structure for each time instance*
- We use two temporal primitives, `before` and `after` (in a strongly reductionist theory one of these would be sufficient)
- *temperature rising* 'temp er temp(before)'
- Transitions are the passing of time (mechanical causation)
- Not a lot of signaling (i/o) across model states, just inertia
- No need to relax CMA limitations

Automata 'living' in model structures

- One state of a model structure can hold a CMA (all its states, transitions, i/o) that runs on a faster timescale
- The internal machine can signal to the larger one, but this takes time to propogate
- No microsynchrony (clock signal) assumed, amorph timing
- Everything finitary/discrete, but it looks as if it was infinite/continuous
- Of what use is your beautiful investigation regarding π? Why study these problems when irrational numbers do not exist? (Kronecker to Lindemann)

Thank You!

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