Algebraic methods in the study of languages and large language models

LaCo Foundational
András Kornai

Abstract

2023 has seen the confluence of the New Minimalist Program (Marcolli, Chomsky, and Berwick, 2023; Marcolli, Berwick, and Chomsky, 2023a; Marcolli, Berwick, and Chomsky, 2023b; Chomsky et al., 2023) and the analysis of Large Language Models by Hopf Algebras (Nemecek, 2023). The goal of this course is to enable non-specialists, both traditional linguists who want to understand the new MP, and computational linguists, logicians, and others interested in LLMs, to approach this emerging body of work.

Lecture 1 Overview of earlier algebraic theories. The AI tradition (Quillian, 1969; Minsky, 1975; Sondheimer, Weischedel, and Bobrow, 1984; Pereira, 2012) used (hyper)graphs. The logic tradition begins with (Ajdukiewicz, 1935; Lambek, 1958; Lambek, 2004) and goes towards Frobenius Algebras (Coecke, Sadrzadeh, and Clark, 2010; Kartsaklis, 2014). The computational tradition (Shieber, 2006; Kornai, 2010; Abend and Rappoport, 2013; Banarescu et al., 2013) explores the connection between term rewriting and machine states (Koller and Kuhlmann, 2012). We outline the conceptual core common to all these in terms of algebra.

Lecture 2 Linear and multilinear algebra. We summarize standard material on vector spaces, matrixes, dimension, rank, inner products, eigenvalues/eigenvectors/eigenspaces, and tensor products. Data compression, PCA, LDA, SVMs. What are word vectors (Schütze, 1993; Collobert et al., 2011), and how to compute them from corpus data.

Lecture 3 Machine learning, deep learning. Classical neural nets (McCulloch and Pitts, 1943; Little, 1974; Smolensky, 1990) and their (linear) algebra. Sequence learning, transformers (Vaswani et al., 2017).

Lecture 4 Foundations of algebra, both classical (set theoretical) and modern (category theoretical). The axiomatic method at the bottom of both. Algebras, bialgebras, Frobenius algebras, Hopf algebras. Automata, transducers, Eilenberg machines.

Lecture 5 AI safety. Alignment, LoRA, safety without alignment (Kornai, Bukatin, and Zombori, 2023)

Kornai is senior scientific advisor at the SZTAKI Computer Science Research Institute and professor at the Department of Algebra, Budapest University of Technology and Economics. He can be reached by mail at MTA SZTAKI, Kende u. 13-17, H-1111 Budapest, Hungary, or by email at andras@kornai.com. His webpage is at kornai.com, that of his students at hlt.bme.hu. He taught *Computational Phonology* at ESSLLI 1992 (Essex), *Mathematical linguistics of pattern recognition* at ESSLLI 2001 (Helsinki), and *Unifying formulaic, geometric, and algebraic theories of semantics* at NASSLLI 2022 (Los Angeles). His latest book is Vector Semantics (Kornai, 2023).

References

Abend, Omri and Ari Rappoport (2013). "UCCA: A semantics-based grammatical annotation scheme". In: *IWCS'13*, pp. 1–12.

Ajdukiewicz, Kazimierz (1935). "Die Syntaktische Konnexität". In: *Studia Philosophica* 1, pp. 1–27.

Banarescu, Laura et al. (2013). "Abstract Meaning Representation for Sembanking". In: *Proceedings of the 7th Linguistic Annotation Workshop and Interoperability with Discourse*. Sofia, Bulgaria: Association for Computational Linguistics, pp. 178–186. URL: https://www.aclweb.org/anthology/W13-2322.

Chomsky, Noam et al. (2023). *Merge and the Strong Minimalist Thesis*. Cambridge University Press. DOI: 10.1017/9781009343244.

Coecke, Bob, Mehrnoosh Sadrzadeh, and Stephen Clark (2010). "Mathematical Foundations for a Compositional Distributional Model of Meaning". In: *arXiv:1003.4394v1*.

Collobert, Ronan et al. (2011). "Natural Language Processing (Almost) from Scratch". In: *Journal of Machine Learning Research (JMLR)*.

Kartsaklis, Dimitrios (2014). “Compositional Distributional Semantics with Compact Closed Categories and Frobenius Algebras”. PhD thesis. Oxford.

Koller, Alexander and Marco Kuhlmann (2012). “Decomposing TAG algorithms using simple algebraizations”. In: *Proceedings of the 11th International Workshop on Tree Adjoining Grammars and Related Formalisms (TAG+ 11)*, pp. 135–143. URL: https://aclanthology.org/W12-4616.pdf.

Kornai, András (2010). “The algebra of lexical semantics”. In: *Proceedings of the 11th Mathematics of Language Workshop*. Ed. by Christian Ebert, Gerhard Jäger, and Jens Michaelis. LNAI 6149. Springer, pp. 174–199. DOI: 10.5555/1886644.1886658.

— (2023). *Vector semantics*. Springer Verlag. DOI: 10.1007/978-981-19-5607-2. URL: http://kornai.com/Drafts/advsem.pdf.

Kornai, András, Michael Bukatin, and Zsolt Zombori (2023). “Safety without alignment”. In: *ArXiv* 2303.00752.

Lambek, Joachim (1958). “The mathematics of sentence structure”. In: *American Mathematical Monthly* 65, pp. 154–170.

— (2004). “A computational approach to English grammar”. In: *Syntax*.

Little, W. A. (1974). “The existence of persistent states in the brain”. In: *Mathematical Biosciences* 19, pp. 101–120.

Marcolli, Matilde, Robert Berwick, and Noam Chomsky (2023a). *Old and New Minimalism: a Hopf algebra comparison*. URL: https://lingbuzz.net/lingbuzz/007373.

— (2023b). *Syntax-semantics interface: an algebraic model*. URL: https://ling.auf.net/lingbuzz/007696.

Marcolli, Matilde, Noam Chomsky, and Robert Berwick (2023). *Mathematical Structure of Syntactic Merge*. arXiv: 2305.18278 [cs.CL].

McCulloch, W.S. and W. Pitts (1943). “A logical calculus of the ideas immanent in nervous activity”. In: *Bulletin of mathematical biophysics* 5, pp. 115–133.

Minsky, Marvin (1975). “A framework for representing knowledge”. In: *The Psychology of Computer Vision*. Ed. by P.H. Winston. McGraw-Hill, pp. 211–277.

Nemecek, Adam (2023). *Coinductive guide to inductive transformer heads*. arXiv: 2302.01834 [cs.LG].

Pereira, Fernando (2012). “Low-Pass Semantics”. In: http://videolectures.net/metaforum2012_pereira_semantic/.

Quillian, M. Ross (1969). “The teachable language comprehender”. In: *Communications of the ACM* 12, pp. 459–476. DOI: 10.1145/363196.363214.

Schütze, Hinrich (1993). “Word Space”. In: *Advances in Neural Information Processing Systems 5*. Ed. by SJ Hanson, JD Cowan, and CL Giles. Morgan Kaufmann, pp. 895–902.

Shieber, Stuart M. (2006). “Unifying Synchronous Tree Adjoining Grammars and Tree Transducers via Bimorphisms”. In: *11th Conference of the European Chapter of the Association for Computational Linguistics*. Ed. by Diana McCarthy and Shuly Wintner. Trento, Italy: Association for Computational Linguistics, pp. 377–384. URL: https://aclanthology.org/E06-1048.

Smolensky, Paul (1990). “Tensor product variable binding and the representation of symbolic structures in connectionist systems”. In: *Artificial intelligence* 46.1, pp. 159–216. DOI: 10.1016/0004-3702(90)90007-M.

Sondheimer, Norman K., Ralph M. Weischedel, and Robert J. Bobrow (1984). “Semantic Interpretation Using KL-ONE”. In: *Proceedings of the 10th International Conference on Computational Linguistics and 22nd Annual Meeting of the Association for Computational Linguistics*. Stanford, California, USA: Association for Computational Linguistics, pp. 101–107.

Vaswani, Ashish et al. (2017). “Attention is All you Need”. In: *Advances in Neural Information Processing Systems 30*. Ed. by I. Guyon et al. Curran Associates, Inc., pp. 5998–6008. arXiv: 1706.03762 [cs.CL]. URL: http://papers.nips.cc/paper/7181-attention-is-all-you-need.pdf.