MCB pg 5

Definition 2.4. *Given a binary non-planar rooted tree* $T \in \mathfrak{T}_{\mathcal{SO}_0}$*, let* $V_{int}(T)$ *denote the set of all internal (non-root) vertices of* T*. For* $v \in V_{int}(T)$*, let* $T_v \subset T$ *denote the subtree consisting of* v *and all its descendants. Let* $L_v = L(T_v)$ *be the set of leaves of* T_v*. The set of accessible terms of* T *is given by*

$$Acc(T) = \{L_v = L(T_v) \mid v \in V_{int}(T)\} . \tag{2.5}$$

For a workspace given by a forest $F = \sqcup_a T_a \in \mathfrak{F}_{\mathcal{SO}_0}$*, the set of accessible terms is*

$$Acc(F) = \bigcup_a Acc(T_a) , \tag{2.6}$$

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MBC pg 18

For a single binary rooted tree T with no assigned planar structure, the set $Acc(T)$ of accessible terms of T is the set of all subtrees $T_v \subset T$ given by all the descendants of a given non-root vertex of T. Indeed, these subtrees T_v are exactly all the intermediate trees obtained in an iterative construction of T starting from the lexical items or features at the leaves, by repeated application of the Merge operation (3.2). We also write $Acc'(T)$ for the set obtained by adding to $Acc(T)$ a copy of T itself, so that

$$Acc'(T) = \{T_v \mid v \in V_{int}(T)\} \quad \text{and} \quad Acc'(T) = \{T_v \mid v \in V(T)\} ,$$

where $V_{int}(T)$ is the set of non-root vertices and $V(T)$ is the set of all vertices of T including the root.

For a workspace given by a forest $F = \sqcup_{a \in \mathcal{I}} T_a \in \mathfrak{F}_{\mathcal{SO}_0}$

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MBC pg 8

Suppose then given two syntactic objects, that is, two $S, S' \in \mathfrak{T}_{\mathcal{SO}_0}$. We define a linear operator

$$\delta_{S,S'} : \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0}) \otimes \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0}) \to \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0}) \otimes \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0})$$

by defining it on generators in the following way. Let

$$\mathfrak{F}^{\Delta}_{\mathcal{SO}_0} = \{(F_1, F_2) \in \mathfrak{F}_{\mathcal{SO}_0} \times \mathfrak{F}_{\mathcal{SO}_0} \,|\, \exists F \in \mathfrak{F}_{\mathcal{SO}_0},\, F_v \subset F \,:\, F_1 = F_v \text{ and } F_2 = F/F_v\} \,. \tag{2.17}$$

For $F_1, F_2 \in \mathfrak{F}_{\mathcal{SO}_0}$, we set

$$\delta_{S,S'}(F_1 \otimes F_2) = 0 \quad \text{for} \quad (F_1, F_2) \notin \mathfrak{F}^{\Delta}_{\mathcal{SO}_0} \,. \tag{2.18}$$

For $(F_1 = F_{\underline{v}}, F_2 = F/F_{\underline{v}}) \in \mathfrak{F}^{\Delta}_{\mathcal{SO}_0}$ with $F = \sqcup_{i \in \mathcal{I}} T_i$, we set

$$\delta_{S,S'}(F_v \otimes F/F_v) = S \sqcup S' \otimes T_a/S \sqcup T_b/S' \sqcup F^{(a,b)} \tag{2.19}$$

with $F^{(a,b)} = \sqcup_{i \neq a,b} T_i$, if there are indices $a, b \in \mathcal{I}$ such that $T_{a,v_a} \simeq S$, $T_{b,v_b} \simeq S'$. If there is more than one choice of indices a, b for which matching pairs $T_{a,v_a} \simeq S$, $T_{b,v_b} \simeq S'$ exist, then the right-hand-side of (2.19) should be replaced by the sum over all the possibilities. We do not write that out explicitly for simplicity of notation. In all other cases (where no matching terms for S and S' are found) we set

$$\delta_{S,S'}(F_{\underline{v}} \otimes F/F_{\underline{v}}) = 1 \otimes F \,. \tag{2.20}$$

MCB pg 9

Definition 2.10. *The action of Merge on workspaces consists of a collection of operators*

$$\{\mathfrak{M}_{S,S'}\}_{S,S' \in \mathfrak{T}_{\mathcal{SO}_0}} \,, \quad \mathfrak{M}_{S,S'} : \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0}) \to \mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0}) \,,$$

parameterized by pairs S, S' of syntactic objects, which act on $\mathcal{V}(\mathfrak{F}_{\mathcal{SO}_0})$ by

$$\mathfrak{M}_{S,S'} = \sqcup \circ (B^+ \otimes \mathrm{id}) \circ \delta_{S,S'} \circ \Delta \,, \tag{2.23}$$

with B^+ the grafting operator of Definition 2.8.

Note that by the definition of $\delta_{S,S'}$ the operator $B^+ \otimes \mathrm{id}$ applied to elements of the form $\delta_{S,S'}(F_{\underline{v}} \otimes F/F_{\underline{v}})$, for $F \in \mathfrak{F}_{\mathcal{SO}_0}$, produces elements $X \otimes Y$ with X in $\mathfrak{T}_{\mathcal{SO}_0}$ and Y in $\mathfrak{F}_{\mathcal{SO}_0}$, hence $\mathfrak{M}_{S,S'}$ maps $\mathfrak{F}_{\mathcal{SO}_0}$ to itself.

The expression (2.23) agrees with the description of the action of Merge on workspaces in [7], [8], namely the Merge operator $\mathfrak{M}_{S,S'}$ searches for copies of the syntactic terms S and S' in the accessible terms of a given workspace F, extracts those accessible terms to perform the Merge operation on, and cancels copies from the workspace, producing the new resulting workspace.

Proposition 2.16. *Consider the modification of* (2.23) *given by*

$$\mathfrak{M}^{\epsilon}_{S,S'} = \sqcup \circ (\mathfrak{M}^{\epsilon} \otimes \mathrm{id}) \circ \delta_{S,S'} \circ \Delta^{(\epsilon,\epsilon^{-1})}\,, \tag{2.26}$$

with $\Delta^{(\epsilon,\epsilon^{-1})}$ *as in* (2.25), *and with*

$$\begin{gathered} \mathfrak{M}^{\epsilon} : \mathcal{V}(\mathfrak{T}_{\mathcal{SO}_0})[\epsilon,\epsilon^{-1}] \otimes_{\mathbb{Q}} \mathcal{V}(\mathfrak{T}_{\mathcal{SO}_0})[\epsilon,\epsilon^{-1}] \to \mathcal{V}(\mathfrak{T}_{\mathcal{SO}_0})[\epsilon,\epsilon^{-1}] \\ \mathfrak{M}^{\epsilon}(\epsilon^{d}\alpha, \epsilon^{\ell}\beta) = \epsilon^{|d+\ell|}\,\mathfrak{M}(\alpha,\beta)\,. \end{gathered} \tag{2.27}$$

Then taking compositions of operations of the form (2.26) *followed by evaluation at* $\epsilon \to 0$ *retains only External and Internal Merge and eliminates all other extended forms of Merge, such as Sideward and Countercyclic.*

Proof. For a single application of (2.26), one obtains terms of the form $\epsilon^{d_v+d_w}\mathfrak{M}(T_v,T_w)$, hence the only terms remaining after taking $\epsilon \to 0$ are of the form $\mathfrak{M}(T,T')$ with T,T' two connected components of the workspace F, which have degree zero in the ϵ variable. These are the External Merge cases. For a composition of two operators of the form (2.26), we regard the result of the first $\mathfrak{M}^{\epsilon}_{S,S'}$ applied to a forest $F \in \mathfrak{F}_{\mathcal{SO}_0}$ as a new workspace, which now carries a dependence on the parameter ϵ. We write such workspaces as $F(\epsilon) = \sqcup_a \epsilon^{d_a} T_a$ in the direct sum (as $\mathbb{Q}$-vector spaces) $\oplus_a \mathcal{V}(\mathfrak{T}_{\mathcal{SO}}\ )[\epsilon,\epsilon^{-1}]$. The composition with a second operator of the form (2.26), then produces terms of the form $\mathfrak{M}(\epsilon^{d}$