Foundations of Mathematics, Lecture 6

András Kornai

BMETE91AM35 Fall 2023-24

Plan of the class

- First 30 minutes: logic
- next 60 minutes: midterm

Logic

- To define a logic we will need four things:
- A language to write formulas
- A notion of truth
- A notion of what the formulas mean 'model theory'
- A deduction procedure 'proof theory'
- We will discuss three main varieties: propositional, first order, and higher order logic
- We will begin at the middle, even though both propositional and higher-order systems are substantially simpler

Rudiments of formal language theory

- Given an alphabet Σ, the set of all strings formed from these is denoted Σ^*. There is a special element λ called the *empty string*.
- Length of λ is 0, length of $a \in \Sigma$ is 1, length of α denoted $|\alpha|$ satisfies $|\alpha\beta| = |\alpha| + |\beta|$
- The main operation on strings is *concatenation* (writing them is sequence). For example, if $\alpha = abc$ and $\beta = AB$ then $\alpha\beta = abcAB$
- Concatenation is *not* commutative, $\beta\alpha = ABabc \neq \alpha\beta$
- We abbreviate $\alpha\alpha$ as α^2, similarly for α^3 etc.
- A **language** over the alphabet Σ is a subset of Σ^*
- Since languages are sets, it is meaningful to speak of their union, intersection, and complement (relative to Σ^*)
- The **product** of languages R and S, written RS, is $\{\alpha\beta | \alpha \in R, \beta \in S\}$
- The set $\cup_{i=0}^{\infty} R^i$ is written R^* and is called the **Kleene closure** of R.