Unifying formulaic, geometric, and algebraic theories of semantics

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NASSLLI Bootcamp Part III

Outline

Intro

- Negation, probability, modality, implicature are all book-length subjects
- They are discussed at chapter length in Chs 4, 5, 6, and 7 respectively in https://kornai.com/Drafts/advsem.pdf
- This lecture will offer a capsule summary of negation and probability, present the layout of the modal system, and leave implicature to the paper https://kornai.com/Papers/truthordare.pdf
- Would like to outline some frontier research problems

Negation: motivation

The key insight: mathematics is not like natural language Benacerraf, 1973:

> *(...) accounts of truth that treat mathematical and nonmathematical discourse in relevantly similar ways do so at the cost of leaving it unintelligible how we can have any mathematical knowledge whatsoever; whereas those which attribute to mathematical propositions the kinds of truth conditions we can clearly know to obtain, do so at the expense of failing to connect these conditions with any analysis of the sentences which shows how the assigned conditions are conditions of their truth.*

NL negation is not an involution, and conjunction is not commutative: *I had dinner and went home* is quite different from *I went home and had dinner*.

Negation in NL

> *(...) the form and function of negative statements in ordinary language are far from simple and transparent. In particular, the absolute symmetry definable between affirmative and negative propositions in logic is not reflected by a comparable symmetry in language structure and language use. Much of the speculative, theoretical, and empirical work on negation over the last twenty-three centuries has focused on the relatively marked or complex nature of the negative statement vis-a-vis its affirmative counterpart. Horn, 1989*

The source of the asymmetry is clear: positive (true) statements are rare, negative (false) statements abound. If I want to describe what I had for breakfast, I cannot list the thousands (millions?) of things I didn't eat, I must make a positive statement.

Negation in the lexicon

About 12% of the 1,200 word defining vocabulary of Release V1 (S19 Appendix 4.8), 144 items altogether, involve some form of negation: *accept accident acid arrive atom bad bar behind bend black block building burn calm catch chance child clean close coal continue continuous cover curve dark dead destroy different dry eager easy elephant end fail finish firm first flat free full gas gradual green hang hard hide ill instead jump laugh leave light limit long lose mean middle must narrow natural necessary need negative new night no nothing object off offensive one only open opinion oppose out park permanent plant police practice preserve prison private protect public quiet reach remove rest right romantic rough rubber rude sad safe same send separate serious sharp short simple sincere single sleep slope smoke smooth soft solid sometimes special steady steal stiff stop straight strange stupid success sudden sure surprise take tent thick thin tie tight together twist unless waste water weak without wrong*

The central observations

- Very well known Marked-unmarked asymmetry. Negative marking often by prefixes like *un-, im-, de-, non-, anti- ...*
- Swept under rug NL negation yields *contraries* not *contradictories*
- Original obs Typically, lexical negation refers to absence of default. People are animals, animals have (functioning) sensory organs, so we don't have to say "seeing person" or "womb-born human", we just say *person, human*. What we need is a special word *blind* which signifies the failure of the default
- Technology We use a binary predicate `lack` for almost all of negation, lexical or syntactic

NO, NOT, N'T

Ordinary (unary) negation is handled by quantifying over the subject variable of `lack` using the one and only quantifier we have, the generic `gen`. In vector semantics, this is simple the d-dim vector that has $1/d$ on each coordinate (and thus unifies with everything).

You shall not kill is represented as `after(gen lack kill)`

`gen` is the same proquant that we use elsewhere to denote a non-specific entity. After the utterance of the command who does no killing? Somebody. Everybody. People. Recipients of the command. It is precisely the generic nature of the subject that guarantees the universal import of the prohibition.

Double negation

Extremely rare in corpora. BNC search reveals 40 examples of *don't don't*, all in live conversation (as opposed to writing), and all with the meaning 'emphatically don't' as in *Charlotte please don't don't go noisy* or *Don't don't you think that there's a conflict of interest there*. This is from a total of 92,334 *don't*s in the corpus. The asymmetry is not restricted to imperatives: consider a grocery store with a sign *no bananas (today)*. Once the shipment arrives, they will not advertise *???no no bananas*.

> *'Natural' negation only involves objects or elements a speaker or listener is attending to ... It makes no sense to instruct a listener to suppress a thought he is not considering or an idea he is not having. De Mey, 1972*

Double negation: the actual cases

- Combine syntactic and morphological negation: *A not unfriendly letter, a not unhappy person . . .*
- Get the same result whether you start with [(not unhappy) person] or [not (unhappy person)].
- First analysis (supported by standard tests of constituency Wells, 1947) begins with `no (gen lack happy)`. One-argument negation negates the main predicate (cf. *John plays/doesn't play golf*. For ¬`lack` we use `has` so we obtain `gen has happy`. When we unify the subject `person` with `gen` we obtain `person has happy`(ness).
- Second analysis begins with `person is_a unhappy` → `person is_a gen lack happy` → `person lack happy`. Negating the main predicate yields `person has happy` again.

Quantifiers

- MG gold standard is to consider these VBTOs, and to use generalized quantifiers (Gärdenfors, 2000; Badia, 2009) for NPs
- Greatest exception on the philosophical tradition is Peirce (see Böttner, 2001 for a modern treatment)
- Form a natural class with pronouns Szabolcsi, 2015
- We model proquants (the pronoun-quantifier class) on nouns and noun phrases: these are just vectors, like all noun-like words
- Consider *somebody, nobody, anybody, everybody; someone, noone, anyone, everyone; somewhere, nowhere, anywhere, everywhere; . . .*
- `who: person, wh; when: time, wh; why: cause_, wh; where: place_ wh; ...`

Probability

Joint work with Zalán Gyenis 

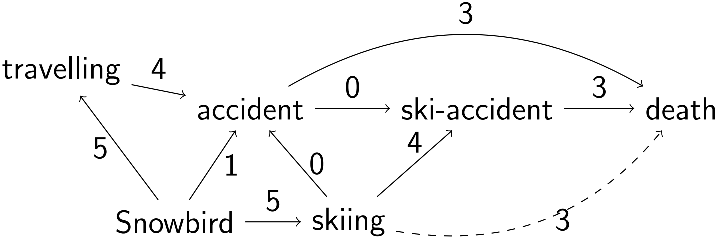
- We use *valuations*, mappings from words to small discrete scales such as *good/neutral/bad*
- For probability, we have a 7-point scale 0=impossible; 1=conceivable; 2=unlikely; 3=neither very unlikely nor very likely; 4=likely; 5 = typical; 6= necessary – somewhat similar to Łukasiewicz L_7
- Naive system lacks additivity $I(A) = \bigoplus I(B_i) = \oplus_{i=0}^{6} \oplus_{I(B_j)=i} i$

Low resolution

- Naive system lacks precision, but so do humans!
- De Méré (1650) problem (solution by Pascal) getting at least one 6 in four rolls of one die ($p = 0.5177$) and getting at least one double-6 in 24 throws of a pair of dice ($p = 0.4914$) (Rényi, 1972; Devlin, 2008)
- Pepys (1690) problem (solution by Newton) at least two 6s when 12 dice are rolled ($p = 0.6187$) and at least 3 6s when 18 dice are rolled ($p = 0.5973$)
- For log odds (Jaynes, 2003) we can use “natural order of magnitude” (Gordon and Hobbs, 2017) to obtain numerical limits

Jeffreys update

$$I(\text{Snowbird} \to \text{travelling}) = 5$$
$$I(\text{Snowbird} \to \text{skiing}) = 5$$
$$I(\text{Snowbird} \to \text{snowing}) = 5$$



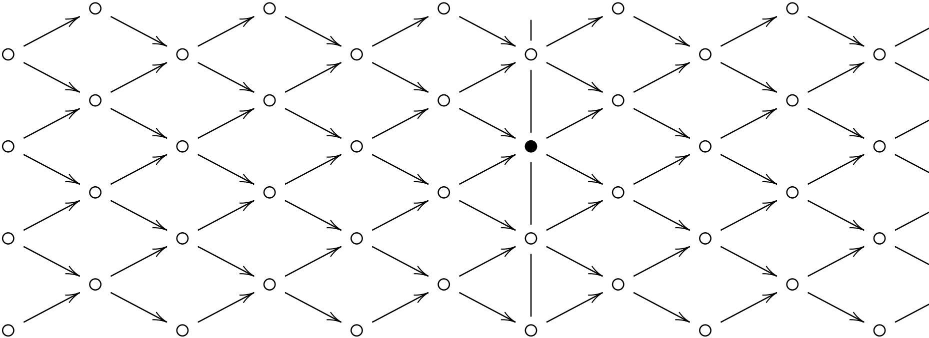
Likeliness of cause of death

Cause of death	Default	Reykjavík	Istanbul	trip
in hospital	4	4	5	4
by accident (non-ski)	4	4	4	5
at home in bed	4	1	1	0
in war	1	0	0	1
by homicide	1	1	1	1
by suicide	2	2	2	1
by forces of nature	1	4	1	2
by ski accident	1	2	1	1

Valuations can model all sort of deductive activity. We assume there is a special 'activity' valuation that is innate: here -1 means 'blocked', 0 means 'inactive', 1 means 'active', and 2 means 'spreading'

Paper on arXiv https://arxiv.org/abs/1905.10924 more details in VS Ch 5

Modal accessibility



The main thrust on one slide

- Terminate the Smolensky program at the quadratic term
- Most words (morphemes) are vectors, only a handful are matrixes, no higher tensors
- In a sparse overcomplete basis we have a few hundred vectors, and we use these to characters not just points in Euclidean space but also the polytopes surrounding them
- Almost all words are definable, each definition an equation
- We can compute the vectors just by solving the system of equations
- Only proper names are points, most words are full polytopes
- Some (12) function words are matrices

Technical summary

- No lexical categories (except for syntax, where they are indispensable) BUT
- Adjectives are typically half-spaces, common nouns and verbs are polytopes, only adpositions/relationals are matrixes
- Subjects are are subsets (local IS_A relations)
- Objects are done by incorporration/vector addition
- Higher arity predication done by decomposition
- Homonymy involves disjoint polytopes, polysemy involves adjacent ones
- Projection mapping is local equality
- Negation is dyadic
- Modalities are valuations

Big wins

- Connecting traditional lexicography to word vectors
- Contexts and thought vectors are matrices manipulating the scalar product
- Unified analysis of *be*
- Clean analysis of thematic roles, intransitive/transitive alt.
- Solution to bunch of traditional problems (negation, quantifiers, implicature)
- A method for deriving the definition of any word in any language
- Explainable entailment Kovács et al., 2022
- Formal semantics for non-compositional cases as well: smooth transition from morphology to syntax

Open research challenges

- (Eilenberg machines) Eilenberg, 1974
- Subdirect decomposition in hypernode graphs
- Tropical geometry of valuations Maclagan and Sturmfels, 2015
- Smooth analysis/synthetic differential geometry Bell, 2008; Moerdijk and Reyes, 1991
- Synchronous parallel rewriting (Alto) (Gontrum et al., 2017)
- `4lang/Reform` Pull requests welcome
- Casualty by temporal succession or by modal alternative?
- Unaccusatives?

Thank you!

Lecture and supporting materials available at
http://kornai.com/2022/NASSLLI

Grand takeaway: sometimes things are *much* easier from one viewpoint than the other.

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