Unifying formulaic, geometric, and algebraic theories of semantics

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ตาบอด
คลำช้าง

Blind men and the elephant

A group of blind men heard that a strange animal, called an elephant, had been brought to the town, but none of them were aware of its shape and form. Out of curiosity, they said: "We must inspect and know it by touch, of which we are capable". So, they sought it out, and when they found it they groped about it. The first person, whose hand landed on the trunk, said, "This being is like a thick snake". For another one whose hand reached its ear, it seemed like a kind of fan. As for another person, whose hand was upon its leg, said, the elephant is a pillar like a tree-trunk. The blind man who placed his hand upon its side said the elephant, "is a wall". Another who felt its tail, described it as a rope. The last felt its tusk, stating the elephant is that which is hard, smooth and like a spear. (Apocryphal back-story to Johnstone, 2002)

Outline

Who is this course for?

- Semanticists, both "mainstream" and "cognitive"
- Morphologists, lexicographers
- Linguists interested in AI, KR, NLP
- People (still) interested in major questions raised 30-50 years ago
- People not afraid of formal theories
- People attracted by some of the coming attractions

Coming attractions

- Brief intro to the five main theory types
- Relatively painless intro to vector semantics
- Foundations of non-compositional semantics
- What do bound morphemes mean?
- Spatiotemporal semantics, ‘projection mapping’, indexicals
- Negation, probability, implicature, modality

Work of many people

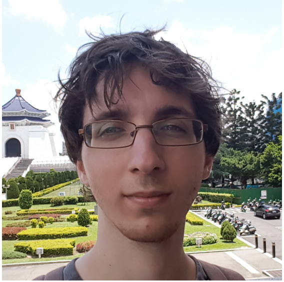
Judit Ács

Márton Makrai

Dávid Nemeskey

Ádám Kovács

Gábor Recski



Dániel Lévai

Format

- Three approx 90 min lectures with two 15 min breaks in between
- Questions taken both during and after lectures
- Readings/slides made available in advance, video afterwards
- Course website at kornai.com/2022/NASSLLI
- Helpful prereq S19 Kornai *Semantics* book https://kornai.com/Drafts/sem.pdf
- Reading: Kornai *Vector Semantics* book https://kornai.com/Drafts/advsem.pdf

The stakes couldn't be higher!

Big tech "giant electronic brains"

The very recent (June 15) https://arxiv.org/pdf/2206.07682.pdf, paper argues that we need ever bigger deep NNs, as these actually show, as we move from zetta (10^{21}) to yotta (10^{24}) scale, emergent properties in semantic performance.

Military "autonomous killer robots"

The very recent (June 1) DARPA ANSR call argues that "data-driven ML lacks transparency, interpretability, and robustness and has unsustainable computational and data needs". The Assured Neuro Symbolic Learning and Reasoning program funds "hybrid AI techniques through relevant military use cases where assurance and autonomy are mission-critical".

Theories of semantics classified by mathematical apparatus

- 1. Logic-based: the Frege–Russell–Tarski–Montague mainstream, henceforth MG (including lineal descendants like Discourse Representation Theory, Dynamic Predicate Logic, Inquisitive Semantics, etc)
- 2. Based on (hyper)graphs: Traditional AI/KR, AMR, 4lang
- 3. Based on linear algebra: distributional semantics (CVS)
- 4. Based on automata theory: Finite State models (operational semantics, see e.g. Fernando, 2018) and S19 Eilenberg machines
- 5. Based on rejection of formal apparatus: cognitive semantics \ Jackendoff

The tusk: logical theories

The mainstream: MG

Attendees are likely to know this, and if not, plenty of great textbooks and advanced courses are available, I recommend Kracht, 2011 and Jacobson, 2014.

Ignoring theory-internal problems, such as hyperintensionals, there are three main issues:

- Logic is too powerful (which makes it unlearnable)
- Meaning postulates are brittle, word meaning remains a mystery
- Creates problems where there are none

The tusk is too sharp

- Problem: logic is not the right tool, it's too sharp. Natural language is incapable of arithmetic
- Everyday language is "a rough and ready instrument incapable of expressing Truth with a capital T" Russell, 1940
- In natural language "it seems to be impossible to define the notion of truth or even to use this notion in a consistent manner and in agreement with the laws of logic" Tarski and Blaustein, 1936 (English tr. 1956)
- "people who put knowledge into computers need mathematical logic, including quantifiers, as much as engineers need calculus" McCarthy, 2005)
- The key takeaway: either you consider *The atomic weight of mercury is 200.592(3)* a natural language sentence, and want a theory that can deal with it, or you are content to consider it a sentence of technical language, outside of scope for natural language semantics, but you can't have both.

The tusk is too weakly attached

The word entropy of natural language is about 12–16 bits/word M08:7.1. Capitalization and punctuation (our best proxies for intonation and related factors) contribute less than 7% (0.12 bits of 1.75 bits per character Brown et al. (1992).

Syntax is an information source of its own. There are many formalisms, we just conider binary trees over n words. These contribute *at most* $\log_2 C_n$ bits. C_n is hard to compute exactly, but asymptotically $C_n \sim 4^n/\sqrt{\pi}n^{1.5}$, so encoding a parse tree requires less than 2 bits/word. (The masoretes used 2 bits for parsing the Bible, Aronoff (1985))

The key takeaway: Information is carried by the words. Logical structure accounts for no more than 12–16% of the information conveyed by a sentence, a number that actually goes down with increased sentence length, and emotive content for even less, perhaps 5–7%.

The tusk is a weapon

Weapons are necessary for certain purposes, but their overuse, actually their very presence, can create problems. When you have a tusk, everything looks like a tree to be debarked.

- Sharp or fuzzy boundaries: are you *fat* when your weight, expressed in kilograms, divided by your height (expressed in meters) squared, is over 30?
- Superfluous readings, "metaphoric usage", and "metonymy" – ordinary language use gets demoted to special status.
- "The first step is to measure whatever can be easily measured. This is OK as far as it goes. The second step is to disregard that which can't be easily measured or to give it an arbitrary quantitative value. This is artificial and misleading. The third step is to presume that what can't be measured easily really isn't important. This is blindness. The fourth step is to say that what can't be easily measured really doesn't exist. This is suicide." (WP on the McNamara fallacy)

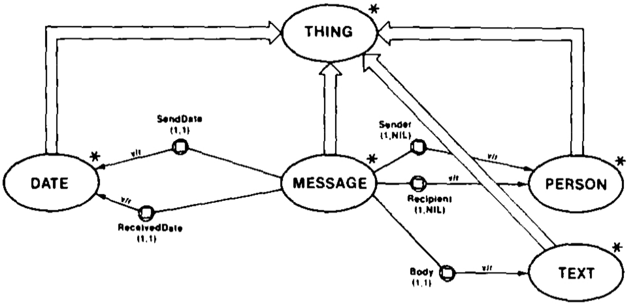
Advantages of truth-theoretical semantics

- Nice clean fun with λ-calculus
- Good (albeit imperfect) stories about intension, modality, temporal reasoning
- Beats the naive "pictures in the mind" theory every possible way
- Good (actually too good) account of quantifiers
- Somewhat good account of pronouns
- Led to the discovery of exciting phenomena (Bach-Peters sentences, paycheck pronouns, non-constituent coordination)
- Fits well with type-theoretical work, programming lg semantics
- Model theory fits well with *reist/concretist* philosophy

The ear: graph representations

- Mainstream approach in AI, its popularity moves in tandem with the AI hype cycle
- Linguists always had their own graphs (constituency, dependency, trees/DAGs, LFG diagrams, . . .)
- Modern, linguistically inspired versions: AMR (Banarescu et al., 2013); `4lang` (Kornai, 2010)
- Now terascale, primary tool in XAI
- Does not require reist underpinnings: there can be ‘real things’ not made of atoms, e.g. feelings, attitudes, circles, . . .

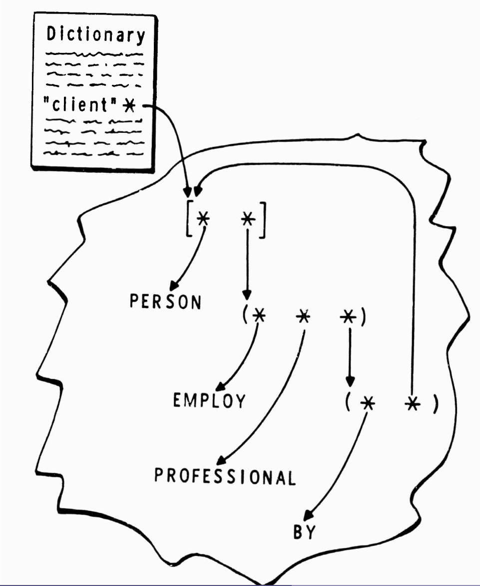
Classic KR



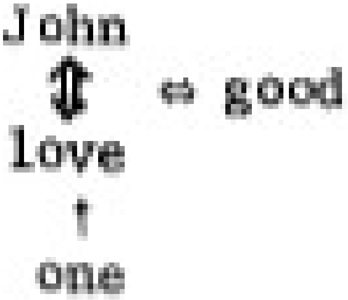
"A MESSAGE is, among other things, a THING with at least one Sender, all of which are PERSONs, at lease one Recipient, all of which are PERSONs, a Body, which is a TEXT, a SendDate, which is a DATE, and a ReceivedDate, which is a DATE."

Quillian, Schank

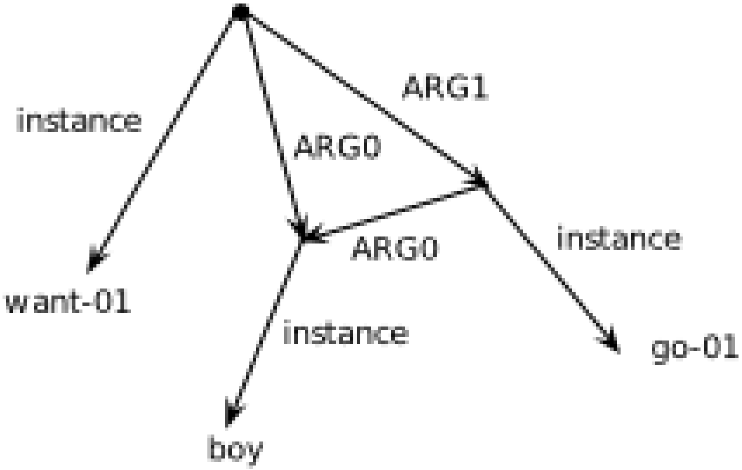
Semantic Memory



Conceptual Dependency



AMR graphs

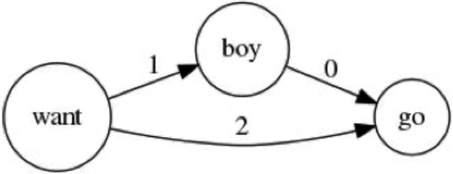


The boy wants to go

AMR graphs *cont'd*

- Rooted, directed, edge- and leaf-labeled graphs
- $\sim$ 100 relations: :accompanier, :age, :beneficiary, :cause, :compared-to, :concession, :condition, :consist-of, :degree, :destination, :direction, :domain, :duration, :employed-by, :example, :extent, :frequency, :instrument, :li, :location, :manner, :medium, :mod, :mode, :name, :part, :path, :polarity, :poss, :purpose, :source, :subevent, :subset, :time, :topic, :value, :quant, :unit, :scale, :day, :month, :year, :weekday, :time, :timezone, :quarter, :dayperiod, :season, :year2, :decade, :century, :calendar, :era
- Neo-Davidsonian graph nodes for entities, events, properties, and states.
- Standardized AMR-parsed corpora (SemBanks) exist for English (60k sentences) and Chinese (5k)

4lang graphs



The boy wants to go

- Have three kinds of links: 0 (is/is_a); 1 (subject); 2 (object)
- In contrast, Cyc has over 45.000 link types, and contemporary efforts like DBpedia or YAGO have $10^5 - 10^6$. The vast majority of these are like *isSpouseOf*, obviously compositional
- `4lang` graphs can be built on RDF-like "triple stores", explicitly addressing known difficulties with these such as **negation, quantifier scope, nested modals** and relations of seemingly **higher arity** *LA is between San Diego and SF along US101*
- Effort to provide semantics for the entire vocabulary

Machine learning on one slide

- Strict separation (typically 80-10-10) of *train*, *dev* and *test* data
- Train is used for building the model, dev for finetuning, test typically hidden from the model builder
- A model optimizes some figure of merit (e.g. word error rate in speech recognition)
- Strong culture of shared tasks (teams working on the same data)
- Generally requires large datasets (gigaword is now typical)
- Supervised methods rule – unsupervised learning still in its infancy
- `aclweb.org/aclwiki/POS_Tagging_(StateOfTheArt)` (see also Manning, 2011)
- For a recent summary see Bengio, LeCun, and Hinton, 2021

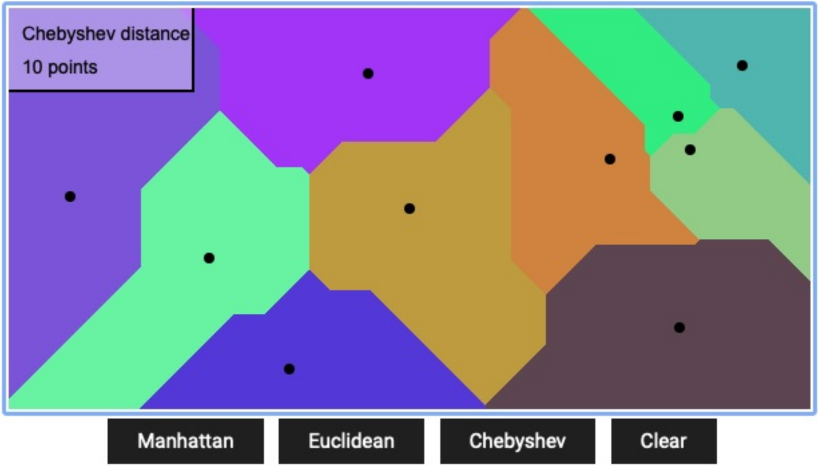
The body: continuous vector space (CVS) semantics

Embedding (static)

Given a dictionary D, a static embeddig is a function $\vec{v}$ that assigns for each word $w \in D$ a vector $\vec{v}(w) \in \mathbb{R}^n$

- First computational treatment by Schütze, 1993 (but goes back to Firth, 1957)
- First implementation that really worked (Bengio et al., 2003)
- NLP "almost from scratch" POS, CHUNK, NER, role labeling (Collobert et al., 2011)
- Has linear structure (king–queen=man–woman) (Mikolov, Yih, and Zweig, 2013)
- Why? (Pennington, Socher, and Manning, 2014; Arora et al., 2015; Gittens, Achlioptas, and Mahoney, 2017)

Voronoi diagram



http://yunzhishi.github.io/voronoi.html

Voronoids

Definition

A *voronoid* $V = \langle \mathcal{P}, P \rangle$ is a pairwise disjoint set of polytopes $\mathcal{P} = \{P_i\}$ in $\mathbb{R}^n$ together with exactly one point p_i in the inside of each P_i.

1. Voronoi diagrams are used in psychological classification (Gärdenfors, 2000). Voronoids are more general, no requirement that
2. the p_i to be at the center of the P_i
3. the facets of the polytopes to lie equidistant from to labelled points
4. the union of the P_i to cover the space almost everywhere – there can be entire regions missing (not containing a distinguished point)

PAC learning + sparsity objective

Linearity

A *linear voronoid* is a voronoid defined by hyperplanes h_j such that every facet of every polytope lies in one of these.

PAC learning

Each concept c corresponds to a probability distribution π_c over $\mathbb{R}^n$

(A concept like *candle* is associated to to other verbal descriptors 'cylindrical, has a wick at the axis, is made of wax, used on festive occasions' and to nonverbal ones, such as a picture of 'the candle' or even the characteristic smell of burning candles.)

We have two objectives: first, to enclose the bulk of each concept set c in some P_i so that $\pi_c(P_i)$ is sufficiently close to 1, and second, to reduce the cardinality of the hyperplane set.

More on word vectors next time

The feet: automata theory

- Operational semantics a la Plotkin/Hennessy "small step" will not be discussed
- This has more to do with the limitations of my understanding than with unworthiness of the approach
- FSTs may get a mention as they are excellent for morphophonological computation
- Eilenberg machines will not be discussed (but see S19:5.8,6.6)
- Will discuss operational aspects for (hyper)graphs and word vectors as we go along
- These are vaguely analogous to "big step" or "natural" semantics a la Kahn, but the analogy will not be exploited

The trunk: cognitive semantics

- Clear linguistic appeal
- Intriguing, but informal, results
- Mainstream formal semantics has nothing to say
- Often insightful, rarely verifiable
- Langacker at the anti-formal extreme, Jackendoff at the formal end, Talmy in between.

Captatio benevolentiae

Textual motivation:

There is in Sullivan's makeup [] an Oxford debater's ready access to the rhetoric of condescending scorn *Jonathan Raban, NYRB 4/12/07*

- What is the extension E (or intension I) of *Sullivan's makeup*?
- Who is *an Oxford debater*?
- Can the *rhetoric of condescending scorn* be analyzed as the genitive of material (just like *a bar of gold*)?
- These concerns are anything but new, see `McCarthy (1976)`

We may not have a full understanding of the relation *x has ready access to y*, but we do know that having ready access to something means that the possessor can deploy it swiftly and with little effort. What the sentence means is simply that Raban finds Sullivan capable of doing so, in fact as capable as those highly skilled in the style of debate practiced at the Oxford Union where condescension and scorn are approved, even appreciated, rhetorical tools.

Long term goals

- characteristica universalis – progress is being made
- calculus ratiocinator – not quite there, especially painful gap in formalizing natural language arguments the way we can formalize mathematical arguments
- Central takeaway from first lecture: Word meaning matters

Recap

- Five classes of models. The body of the elephant (by far the largest volume, with thousands of researchers using continuous vector space semantics) is vector semantics.
- Why is this the body? Because word meaning carries the bulk of the information, over 80%.
- Why not discrete vectors? Binary feature vectors work well in phonology and morphology, and theories capturing word meaning in terms of simple structures (trees) built from these have been around at least since Katz and Fodor, 1963.
- Yes, but their learning theory is weak. Using continuous vectors gives us differentiability, differentiability gives us gradient optimization, gradient optimization can be used for learning.

Terminology

"Embedding" is just another word for word vectors, standardly defined as a mapping (context-free in the static case, context-sensitive in the dynamic case) of a dictionary to $\mathbb{R}^n$.
"Distributional semantics" is just another word for the key idea for creating the mapping, *You shall know a word by the company it keeps* (John Rupert Firth)

Neural "brain" models

- Long history, with mathematical models going back to McCulloch and Pitts, 1943, Rosenblatt, 1957
- We are more interested in the mathematical side than in actual brain science (Hertz, Krogh, and Palmer, 1991)
- Minsky and Papert, 1988 (3rd ed, originally 1969) unrepentant in their dismissal of neural nets. See Pollack, 1989 for a discussion.
- In the history of ideas, the XOR and parity issues were a real problem, unsolvable by the classic single-layer perceptron.
- Remarkably, the solution, multi-layer NNs and backprop, was found as early as Bryson and Ho (1969), but not fully appreciated until Werbos (1974), Parker (1985), Le Cun (1985), and Rumelhart, Hinton, and Williams, 1985.

Parity

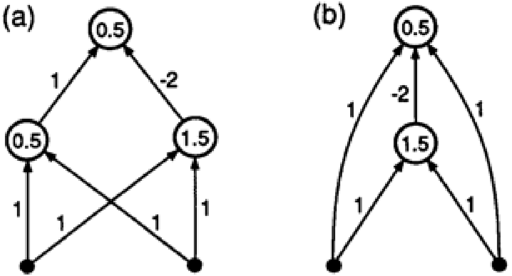


FIGURE 6.5 Two networks that can solve the XOR problem using 0/1 threshold units. Each unit is shown with its threshold.

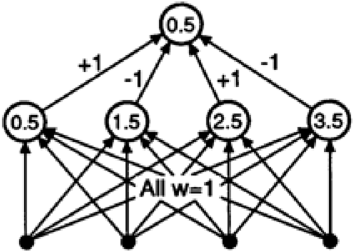


FIGURE 6.6 A network to solve the $N = 4$ parity problem with 0/1 threshold units.

Takeaways from figure

Sutskever et al 2014

large DNNs can be trained with supervised backpropagation whenever the labeled training set has enough information to specify the network's parameters. Thus, if there exists a parameter setting of a large DNN that achieves good results (for example, because humans can solve the task very rapidly), supervised backpropagation will find these parameters and solve the problem.

- Discrete system is embedded in continuous one (Gyenis, 2018)
- It is the continuous aspects that enable gradient learning
- We will start the analysis based on Little, 1974
- Single layer, but recurrent (contains multilayer as special case)
- Bra-ket notation, the computation already done in Ashkin and Lamb (1943), relevant math goes back to early 20th c.
- Please, no "brain haz quantum" amateur philosophy!

Setup

- n binary neurons (± 1, using $0/1$ would make no difference)
- A state is fully characterized by a *thought vector* $\Psi(t) = |s_1, \ldots, s_n\rangle$.
- Connection strengths are given by $n \times n$ matrix V
- Sigmoid activation function $\sigma_\beta(r) = \frac{1}{1+e^{-\beta r}}$
- Incoming activation on i is $r = \Sigma_j V_{ij} \frac{s_j+1}{2}$
- Probability of neuron i fireing at t is

$$\sigma_\beta(\Sigma_j V_{ij} \frac{s_j + 1}{2} - V_0)$$

The biggest matrix you have ever seen

- With $n \sim 10^{11}$ neurons, the thought vector is BIG.
- Some of the positions may be *clamped* to -1 or $+1$ by external (sensory) or internal (proprioceptory) input
- Left alone, the thought vector follows a path on the n-dimensional hypercube determined by a 2^n by 2^n transition matrix P that defines the scalar product $\langle \Psi(t+1)|P|\Psi(t)\rangle$
- P is changing adiabatically (on the order of seconds or even hours) relative to the state vector changes (microsecond range), so we assume it's fixed (no learning, no senescence)
- Let ϕ_r be the unit length eigenvectors of P corresponding to eigenvalues λ_r (initially all assumed different) and express Ψ in this basis as $\psi(\Psi) = \sum_r \phi_r(\Psi)$. Since the eigenvalues are different (with probability 1) the eigenvectors are orthogonal, so the scalar product simplifies to

$$\langle \Psi(t+1)|P|\Psi(t)\rangle = \Sigma_r \lambda_r \phi_r(\alpha(t+1))\phi_r(\alpha(t))$$

Temporal evolution

- Time average $\Gamma(\alpha)$ of the probability of the system being in state α is
$$\Gamma(\alpha) = \frac{\sum_r \lambda_r^M \phi_r^2(\alpha)}{\sum_r \lambda_r^M}$$
- If there is a unique largest eigenvalue λ_1, for large M the contributions of all the other eigenvectors and eigenvalues will be negligible both in the numerator and the denominator, so the sum reduces to
$$\Gamma(\alpha) = \phi_1^2(\alpha)$$
- When there exist two or more largest eigenvalues λ_1 and λ_2, with corresponding eigenvectors ϕ_1 and ϕ_2, we obtainin
$$\Gamma(\alpha, \beta) = \frac{\lambda_1^M \phi_1^2(\alpha) + \lambda_2^M \phi_2^2(\alpha)}{\lambda_1^M + \lambda_2^M}$$
- In general, we have $\Gamma(\alpha, \beta) = \phi_1^2(\alpha)\phi_1^2(\beta) = \Gamma(\alpha)\Gamma(\beta)$

Temporal evolution *cont'd*

In general, the long term probability distribution of β is totally uncorrelated to that of α after a large number of steps. Little, 1974 interprets this as the system being largely incapable of having persistent states, and only if λ_1 and λ_2 are sufficiently close can we

> have the possibility of states occurring (...) which are correlated over arbitrarily long periods of time. It is worth noting too that the characteristics of the states which so persist are describable in terms of the eigenvectors associated only with the degenerate maximum eigenvalues. In this sense these persistent states are very much simpler to describe than an arbitrary state (...) for they involve only that small set of eigenvectors associated with the degenerate maximum eigenvalues, whereas other states (require) the full set of 2^n eigenvectors.

Word vectors are in the persistent subspace

- Takeaway so far: there are gazillions of thought vectors ($2^{10^{11}}$), but only a small fraction of these will be persistent. These are the ones expressible as linear combinations of the eigenvectors corresponding to the maximal and near-maximal eigenvalues.
- We will look at the *persistent linguistic subspace* of the thought space, assuming it is spanned by some eigenvectors $p_1, \ldots, p_d$. Assuming a working lexicon of $10^4 - 10^6$ entries, d is less than a million, even if these lexical entries would all be independent
- In fact, we will argue that $\dim(\mathrm{L}) < 10^3$. The word vectors generally used in practice have $300 \leq d \leq 800$.
- Our *representation space* will be $R = L \times L$ and a few copies of this

How do we compute word vectors

- Two main classes: static, and dynamic (context sensitive)
- We will discuss chiefly static, but *multi-sense* embeddings, where each word sense gets its own vector
- But first, a taste of the dynamic embeddings, in this case GPT-2. You can try it yourself at `https://talktotransformer.com` by entering any initial seed and see what continuation it generates. We take our seed from the NYT, 06/25/20, *Trump Overhaul of Campus Sex Assault Rules Wins Surprising Support* by Michael Powell.

Education Secretary Betsy DeVos fired a shot last month in the nation's culture wars, overhauling how colleges handle investigations of sexual assault and ending what she called Obama-era "kangaroo courts" on campus.

She is again laying out her plan. The goal: create an independent "office of gender equity" with prosecutorial authority to give victims of sexual assault "a complete range of rights, services, and protections." But the institute with the responsibility for enforcing the new policies will be created by the U.S. Department of Education, not by the Justice Department, her department said. That was a break from President Donald Trump, who gave the independent office the word "Attorney General."

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That is no small move. The Obama administration's response to campus sexual assault has been tepid at best and haphazard at worst, reflecting administration overreach. Some states were given oversight of universities by the Education Department for handling the investigation, but the Obama administration did little to ensure universities complied. The new guidance begins to address those shortcomings. The new Title IX guidance centers on the extent to which schools have a responsibility to the victim of sexual assault and to any witnesses to the crime. Under the Obama guidance ...

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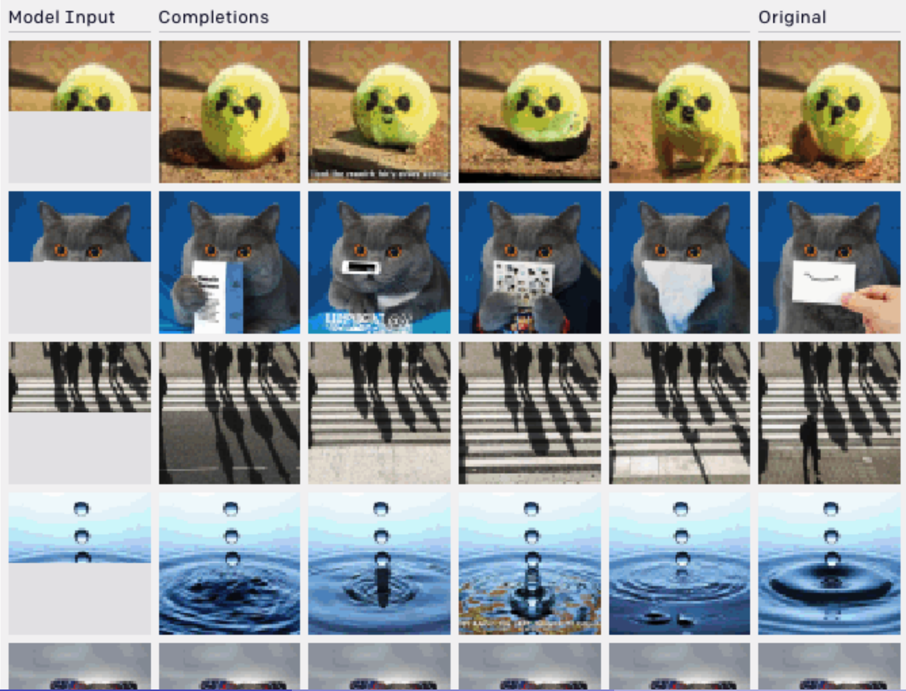
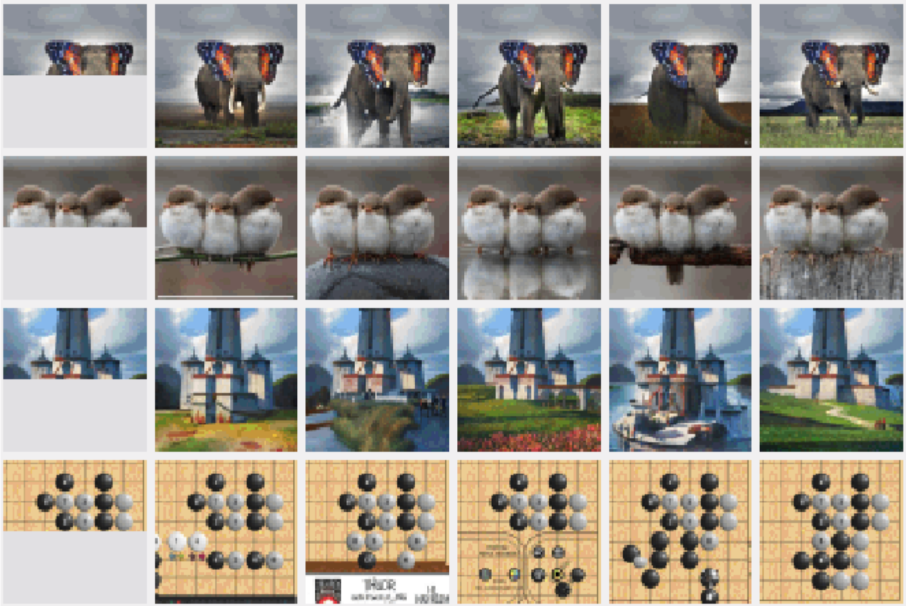
The new Education Department rules give more protections to the accused, primarily young men who face discipline or expulsion as a result of allegations of sexual misconduct. The move set off a liberal uproar, denounced by unions representing teachers and college professors, by the National Organization for Women and by an array of Democratic senators. The Trump rules, they said, constitute a radical rollback of protections for victims who seek justice after sexual assaults.


Model Input
Completions
Original



Some unpleasant takeaways

- The grammar is good. Yet there is no overt rule of subject-predicate agreement, there are no rules, no constraints, no principles
- Generative grammar is epiphenomenal
- Any kind of appeal to UG/LAD seems misplaced
- Appeal to genetic structure is massively misplaced
- By its own measures, CL is only about 55% done
- eXplainable AI (XAI) is a key goal

Predecessors to word vectors: PCA

- Typical survey questions ask us to rank actions, objects, or statements on a scale: snails au gratin are very appetizing (+2); somewhat appetizing (+1); neither appetizing nor disgusting (0); somewhat disgusting (−1); or very disgusting (−2).
- Assume you have obtained a total of N responses from r respondents to n questions about m items: this can be summarized in a 3D array S whose (i, j, k) element is the response of respondent i to question j about object k
- Principal Component Analysis (PCA) Pearson, 1901: (a) normalize to 2d array: ignore the j, k structure and collect all responses by a given subject i in a row vector (with nm coordinates); collect these vectors in a data matrix D with r rows and $c = nm$ columns. (b) normalize the data by substracting the mean of each column from each entry in that column (means centering)

PCA *cont'd*

- $D^T D$ gives the *covariance matrix* C which has size $r \times r$, is symmetrical, and positive semidefinite. The variance in an arbitrary direction $\vec{x}$ is given by $\vec{x}^T C \vec{x}$, and the first principal component of the data is defined as the direction that maximizes the variance. To find it, we need to solve

$$\frac{d}{d\vec{x}} \vec{x}^T C \vec{x} - \lambda \vec{x}^T \vec{x}$$

  (where the second term is the Lagrange multiplier that comes from the constraint of keeping the length of $\vec{x}$ fixed)
- The critical points are obtained from solving $C\vec{x} = \lambda\vec{x}$, so the solutions λ_i are by definition the eigenvalues, and the x_i are the corresponding eigenvectors
- As always, the eigenvector basis is the winner

SVD reformulation

- Let the Singular Value Decomposition (SVD) of D be UGV^T. The columns of V are exactly the eigenvectors of C, and the positive singular values found in the diagonal matrix G (conventionally arranged to run from larger to smaller) are the square roots of the eigenvalues λ_i of C, which we use to measure the "goodness" of principal components. Writing $\Lambda = \sum_{i=1}^{c} \lambda_i$, we say, slightly misleadingly, that each λ_i *accounts for* a fraction λ_i/Λ of the total variance.
- By the Eckart–Young theorem, if the first a columns of U are collected together in U_a, the first a columns of V in V_a, and the first a singular values (by decreasing size) in G_a, the matrix $C_a = U_a G_a V_a^T$ is the best rank-a approximation (in Frobenius norm) of C. This approximation is unique as long as the first a eigenvalues are distinct, a condition generally met in the cases of interest.

Key takeaways

- Geometric intuition is nice: *dog* is closer in meaning space to *cat* than to *harpsicord*. But the key advantage of vectors is not 3d intuition, it is the apparatus (vectors, matrices, norms, eigenvalues, ...) that lets you compute things!
- Data compression is key: just as keeping the first few terms of a Taylor series is usually a good approximation strategy, keeping the first few eigenvectors provides a good approximation (often the best possible)
- Data is always noisy!
- Surveying people is an expensive, error-prone process, the existing datasets (WordSim-353, SimLex-999, MEN) are only used for testing, not training
- Early applications like Osgood, May, and Miron, 1975 involved 100x100 matrices ($10^4 - 10^5$ matrix elements)

Predecessors to word vectors 2: LSA

- Latent Semantic Analysis Deerwester, Dumais, and Harshman, 1990 ignores what people say about relatedness, observes their behavior instead. The assumption is that they will use similar words in similar documents.
- We create a *term-document matrix* T which counts how often word i appears in document j
- We transform the entries e.g. by using

$$a_{ij} = (\log T_{ij} + 1)(\sum_j p_{ij} \log p_{ij} / \log n + 1)$$

- Looks like "secret sauce", really just entropy-based normalization
- We apply SVD, keep only the first few hundred eigenvectors
- Development held back for many years by patents and compute issues (10k+ terms, 100k+ documents, 10^9 matrix elements)

Skip-Grams with Negative Sampling (SGNS)

- Assume a gigaword corpus (word sequence) $w_1, \ldots, w_n$. The *context* of length L for word w_i are the words $w_{i-L}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+L}$
- Our data D are the observed word-context pairs
- We want to assign word vectors w and context vectors c so that the probability that $(w, c) \in D$ is modeled by $\sigma(< w, c >) = \frac{1}{1+e^{-<w,c>}}$ where σ is the usual "sigmoid squishing" used in neural nets
- We maximize $\log \sigma(< w, c >)$ for observered pairs, and use k "negaive samples" (pairs not in D) to maximize $k \log \sigma(< -w, c_N >)$ where the c_N are simply drawn randomly from the distribution of the contexts – we assume that a random context is unlikely to fit w
- These models came early Mikolov et al., 2013, and are still very useable. Efficient implementations exist

Implicit factorization

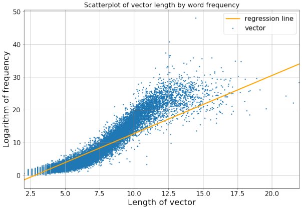
- We pretend to build a term-context matrix as we built the term-document matrix for LSA
- But we have 50-100k words, gw corpus (10^{14} matrix elements)
- We have an 'implicit matrix' whose element (w, c) measures the strength of the association between word w and context c using pointwise mutual information

  $$\log \frac{\#(w, c)D}{\#(w)\#(c)} - \log k$$

  where D is corpus size and k is the degree of negative sampling Levy and Goldberg, 2014
- There are many technical tricks: using only the positive values of PMI (PPMI), using just the eigenvectors without the eigenvalues, etc etc.
- Methods built on direct optimization of difference between predicted and observed association such as `word2vec` came first, and are still very useable Mikolov et al., 2013

Fundamental properties of word vectors

1. Frequency



$$\log(p(w)) = \frac{1}{2d}||\vec{w}||^2 - \log Z \pm o(1) \tag{1}$$

2. Cooccurrence estimate

$$\log p(w, w') = \frac{1}{2d}||\vec{w} + \vec{w'}||^2 - 2\log Z \pm o(1) \tag{2}$$

3. PMI

$$\langle \vec{w}, \vec{w'} \rangle \sim \frac{\log p(w, w')}{\log p(w) \log p(w')} \tag{3}$$

Why the additive structure?

- Mikolov, Yih, and Zweig, 2013 noted *king–queen=man–woman*. Analogical puzzles like *Steve Jobs is to Apple as Bill Gates is to X* are readily solved by computing the vector Apple+Gates–Jobs and searching for the nearest vector in the embedding. This also works for morphology: not only is *boy–boys=goat–goats* but also = *mouse–mice*.
- Why? Four explanations. Pennington, Socher, and Manning, 2014 suggests

$$\frac{p(C|king)}{p(C|queen)} \approx \frac{p(C|man)}{p(C|woman)}$$

i.e. that the conditional probability of most contexts (e.g. *water*) is generally independent of the choice between *king* or *queen*, *man* or *woman*, and the ratios will deviate exactly for the same contexts like *dress, he, she, Elizabeth, Henry* ...

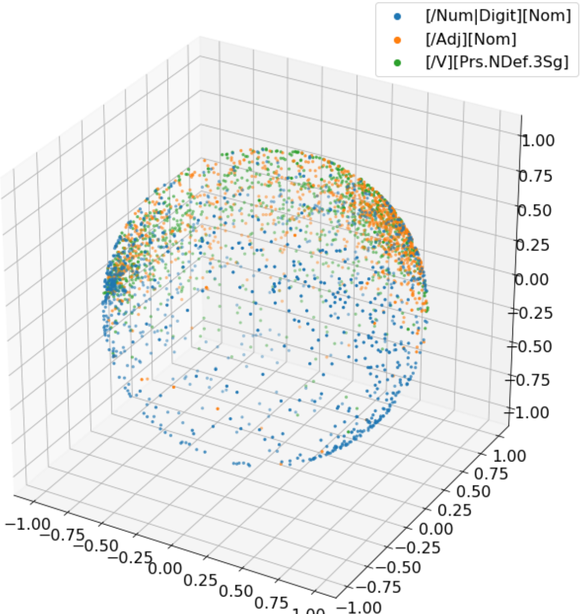
Additive structure *cont'd*

- Levy and Goldberg, 2014 Suggests essentially the same, assuming
$$< w, C > \approx \log \frac{p(w, C)}{p(w)p(C)}$$
- Arora et al., 2015 Embeddings are *approximately isotropic* meaning $\mathbb{E}_w < w, w >$ is approximately the identity matrix in the sense that all its eigenvalues lie in the $[1, 1+\delta]$ interval for some small δ. If so, $\text{argmin}_d ||a - b - c + d||_2^2$ is $\approx$ $\text{argmin}_d \mathbb{E}_w < a, w > - < b, w > - < c, w > + < d - w >$ which goes back to the same goal of finding a word w that will minimize
$$\sum_C \log \frac{p(C|king)}{p(C|queen)} - \log \frac{p(C|man)}{p(C|w)} \qquad (4)$$
where the sum is taken over all contexts C.

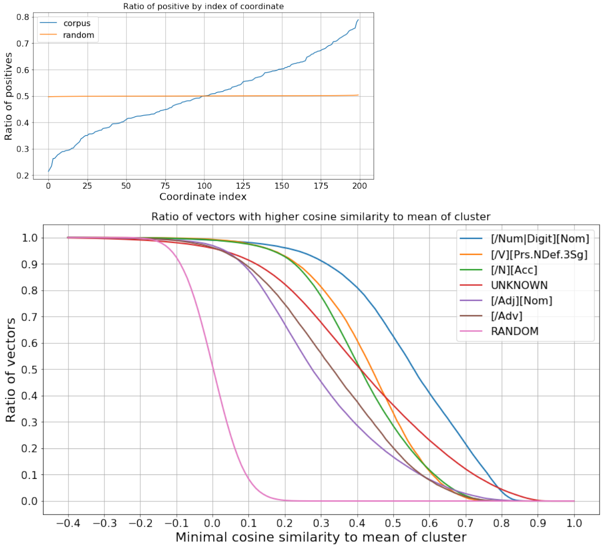
Additive structure *cont'd*

- Finally, the brilliantly titled Gittens, Achlioptas, and Mahoney, 2017 **Skipgram − Zipf + Uniform = Vector additivity** analyzes the original SGNS model and concludes that, with the assumption of uniform, rather than Zipfian frequency distribution, it is equivalent to the Sufficient Dimensionalty Reduction model of Globerson and Tishby, 2003, and will be even more additive in the sense that context vectors are simply the addition of the word vectors that appear in the context!
- None of these explanations are built entirely on realistic assumptions: context vectors are not random walks (Arora et al), frequency distributions are not uniform (Gittens et al), and there are more subtle but discernible problems with the Levy and Goldberg and the Pennington et al explanations as well. Yet the phenomenon is real, additivity is a thing. The puzzle is solved 75% of the time, but see Nissim, Noord, and Goot, 2020 for a major trick, without which we only get 45%.

More on (static) geometry



Geometry very far from random

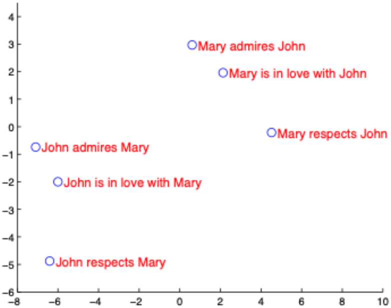


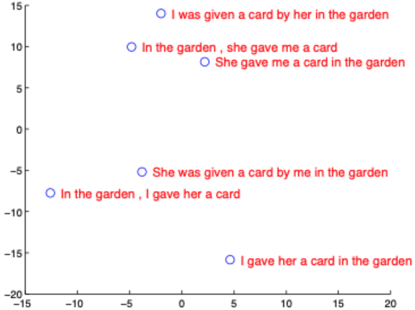
(Data from Lévai and Kornai, 2019)

Seq2seq

- Sequence to sequence (seq2seq) tasks are commonly seen in machine translation, named entity recognition, POS tagging, etc. In the *encoder* stage we present the network with a sequence of inputs (word vectors) and it maintains state in a single, fixed length state vector. When a special EOM token is presented, the network moves in a *decoder stage* and uses the state vector (and in subsequent steps, its previous output) to generate new words, until it generates an EOM.
- Sutskever, Vinyals, and Le, 2014 first demonstrated that this works well in MT, especially if the source lg sequence is presented in reverse order during training. They used LSTMs with 4 hidden layers for encoders and decoders, 160k source lg words, 80k target lg words, plus the UNK token.
- At time t, the network performs $h_t = \sigma(W^{hx}x_t + W^{hh}h_{t-1})$ and $y_t = W^{yh}h_t$, where W^{ij} is the connection strength matrix from i to j.

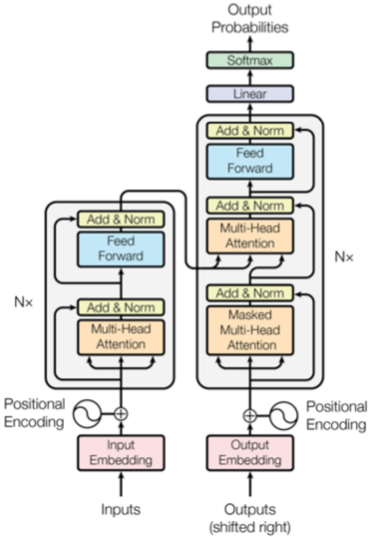
State vectors for different sentences





Transformer

Built on the idea of removing the recurrent aspects of seq2seq by replacing temporal behavior by spatial connections called *attention*
Vaswani et al., 2017



Transformer descendants

- Katharopoulos et al., 2020 defines *autoregressive* transformers, bringing back the temporal (recurrent) view
- BERT is a transformer model, using "wordpiece" vocabulary
- GPT-2 doesn't use a decoder Radford et al., 2019
- Currently at the top of the hype cycle (thousands of papers/year)
- Reproducibility crisis

15 min break

2nd part: (hyper)graphs, lexicon, non-compositionality. Morphology and lexicography with vectors.

3rd part: negation, modality, probability

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