

A MATEMATIKA ALAPJAI, 7. ELŐADÁS

Kornai András

BMETE91AM35 2022-23 Őszi Félév

- Házik megbeszélése
- 20 perc próbázh
- ZH 2 hét múlva, november 2. 24 órán belül be lehet adni ugyanazt latexben is extra pontokért
- Logika indul

LOGIC

- The twin pillars of the foundation are set theory and logic
- We now turn to logic
- Set theory has variants (ZFC, NGB, KP . . .), logic has many more variants!
- To define a logic we will need four things:
 - ① A language to write formulas
 - ② A notion of truth
 - ③ A notion of what the formulas mean 'model theory'
 - ④ A deduction procedure 'proof theory'
- We will cover each in turn, starting with 'language'
- We have a set Σ called the *alphabet*, it's elements are called *letters*
- Putting letters one after the other we obtain *strings*

RUDIMENTS OF FORMAL LANGUAGE THEORY

- Given an alphabet Σ , the set of all strings formed from these is denoted Σ^* . There is a special element λ called the *empty string*.
- Length of λ is 0, length of $a \in \Sigma$ is 1, length of α denoted $|\alpha|$ satisfies $|\alpha\beta| = |\alpha| + |\beta|$
- The main operation on strings is *concatenation* (writing them in sequence). For example, if $\alpha = abc$ and $\beta = AB$ then $\alpha\beta = abcAB$
- Concatenation is *not* commutative, $\beta\alpha = ABabc \neq \alpha\beta$
- We abbreviate $\alpha\alpha$ as α^2 , similarly for α^3 etc.
- A **language** over the alphabet Σ is a subset of Σ^*
- Since languages are sets, it is meaningful to speak of their union, intersection, and complement (relative to Σ^*)
- The **product** of languages R and S , written RS , is $\{\alpha\beta \mid \alpha \in R, \beta \in S\}$
- The set $\cup_{i=0}^{\infty} R^i$ is written R^* and is called the **Kleene closure** of R .