A matematika alapjai, 6. előadás

Kornai András

BMETE91AM35 2022-23 Őszi Félév

Ezen az órán

- Dolgozunk a házi feladatokon
- Műveletek, struktúrák
- A ZH a 9. héten (nov 2) lesz!

Műveletek

- We know many kinds: operations on numbers, operations on sets, operations on functions, on logical formulas, etc etc
- The most typical case is *binary*: take two inputs X and Y, and produce a single output $X + Y$, $X \cup Y$, $X \circ Y$, $X \vee Y$, ...
- Binary operations are functions with two variables
- *Unary* operations are like negative $-X$, reciprocal X^{-1}, negation, etc. These are functions with one variable i.e. binary relations i.e subsets of S^2
- *Nullary* operations are functions with no variable, i.e. *constants*, one-member subsets of S^1. Why one-member (singleton)? Because the output is unique.
- Typical examples are "distinguished elements" such as arithmetic 0 or 1, the truth values, etc.
- There can be higher *arity* operations, e.g. tern*ary* (function with 3 variables) etc. There can also be operations involving elements from more than one structure

Structures

- A structure is a set called the 'base set' endowed with some operations (typically only finitely many) each with a given arity.
- These must satisfy the axioms that characterize the structure
- Well-known examples: groups, rings, fields, Boolean algebras
- We start with *semigroups* which have one binary operation satisfying $(ab)c = a(bc)$ (associativity)
- If we add one nullary operation e that satisfies $ea = ae = a$ we have a *monoid*
- If we also add a unary operation $^{-1}$ that satisfied $aa^{-1} = e$ we have a *group*

Group theory: this is where the fun begins

- The *order* of a group is the cardinality of its base set $|G|$
- The *order* of an element a is the smallest n for which $a^n = e$
- Theorem (Cauchy) in a finite group the order of any element divides the order of the group HW: try to prove this on your own
- Generators