

A MATEMATIKA ALAPJAI, 11. ELŐADÁS

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PROOF THEORY

- 1 The goal: mechanical checking of proofs
- 2 We will manipulate proofs by elementary steps
- 3 Just one example: Gentzen 'natural deduction' and 'sequent calculus'
- 4 Every step fits the form $A_1, \dots, A_n \vdash B_1, \dots, B_k$ The sides are called *sequents*. **WARNING: Left side interpreted conjunctively (',' means \wedge), right side interpreted disjunctively (',' means \vee)**
Natural deduction is the special case when $k = 1$
- 5 Why is this a good trick? Because implication $A \rightarrow B$ means $\neg A \vee B$ so when we move stuff to the other side it changes "sign" as in arithmetic. Just one axiom $p, r \vdash q, r$
- 6 Manipulation rules are written above and below a horizontal line

e.g. $L \wedge$ rule:

$$\frac{\Gamma, A \wedge B \vdash \Delta}{\Gamma, A, B \vdash \Delta}$$

Left:

Right:

$$L \wedge \text{rule: } \frac{\Gamma, A \wedge B \vdash \Delta}{\Gamma, A, B \vdash \Delta}$$

$$R \wedge \text{rule: } \frac{\Gamma \vdash \Delta, A \wedge B}{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}$$

$$L \vee \text{rule: } \frac{\Gamma, A \vee B \vdash \Delta}{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}$$

$$R \vee \text{rule: } \frac{\Gamma \vdash \Delta, A \vee B}{\Gamma \vdash \Delta, A, B}$$

$$L \rightarrow \text{rule: } \frac{\Gamma, A \rightarrow B \vdash \Delta}{\Gamma \vdash \Delta, A \quad \Gamma, B \vdash \Delta}$$

$$R \rightarrow \text{rule: } \frac{\Gamma \vdash \Delta, A \rightarrow B}{\Gamma, A \vdash \Delta, B}$$

$$L \neg \text{rule: } \frac{\Gamma, \neg A \vdash \Delta}{\Gamma \vdash \Delta, A}$$

$$R \neg \text{rule: } \frac{\Gamma \vdash \Delta, \neg A}{\Gamma, A \vdash \Delta}$$

MODEL THEORY

- We build *model structures* as sets endowed with the right kind of relations
- For example, a model of the natural numbers is a set M endowed with a succession relation $s \subset M \times M$ and satisfying the Peano axioms
- An *interpretation* is a mapping from formulas to a model
- Key notion: *elementary equivalence* holds between structures if they satisfy the same FOL sentences
- Lowenheim-Skolem theorem: if a theory has an infinite model, it has a countably infinite model ‘downward L-S’
- If it has a countable model, it has a model at any cardinality ‘upward L-S’

HOMEWORK, FINAL

HW11.1-5 are CPZ 11.3; 11.5; 11.13; 11.21; and 11.31 respectively.

Final December 7 at 8:30 (usual time and place). This will cover Chapters 1-11 of CPZ (first 303 pages)