Dynamical data analysis: Unicorns, causes and consequences

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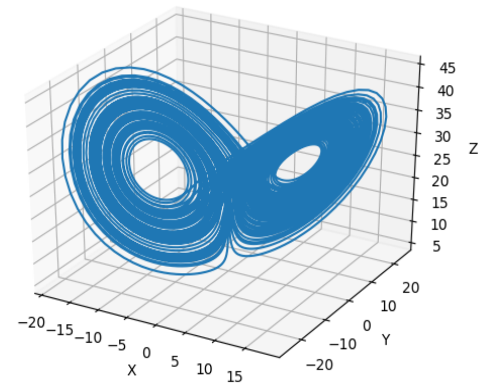
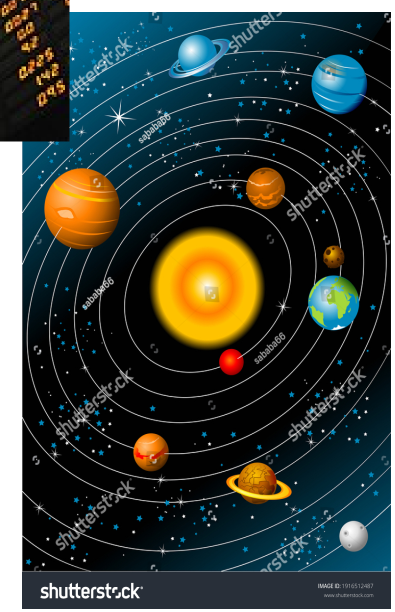
Nonlinear dynamical systems

- The System: the object of the investigation
  - Eq.: a patient, a computer, stockmarket
- Variables:
  - Bood pressure, temperature, position, velocity, bits, stock prices
- State:
  - A set of variables in a moment that determines the temporal evolution of the system
- State space
  - All the possible states
- Evolution
  - Update rules ...



http://www.flickr.com/photos/thewalkingirony/3051500551/ | Author=Katrina.Tuliao



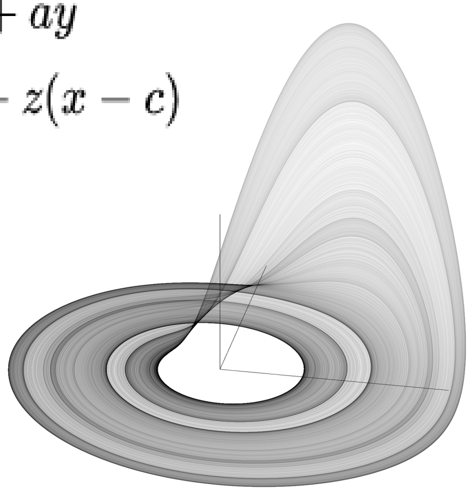
Nonlinear dynamical systems

Contiuous-time dynamics

$$\dot{s} = \varphi(s)$$

- Rössler system

$$\frac{dx}{dt} = -y - z$$
$$\frac{dy}{dt} = x + ay$$
$$\frac{dz}{dt} = b + z(x - c)$$

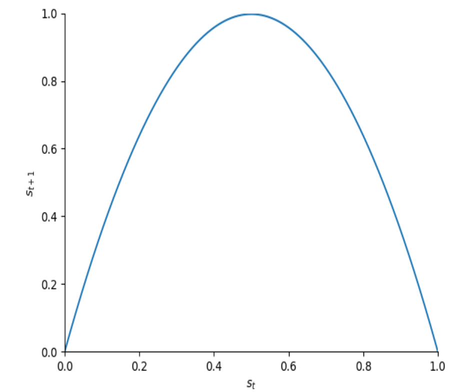


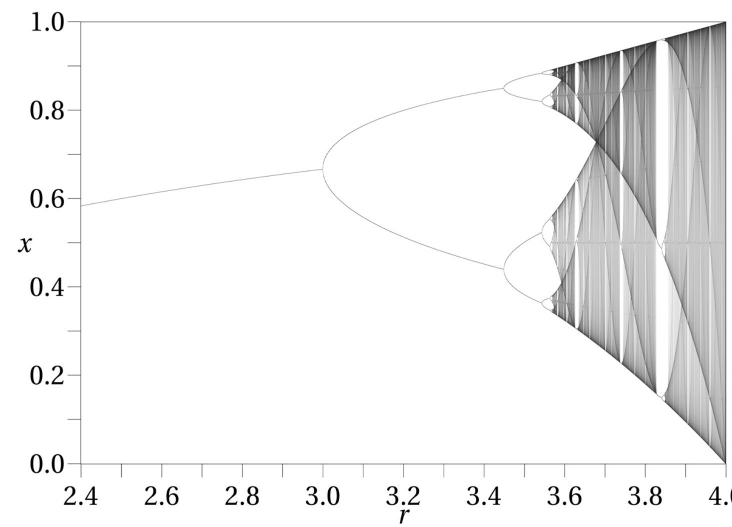
Discrete-time dynamics

$$s_{t+1} = f(s_t)$$

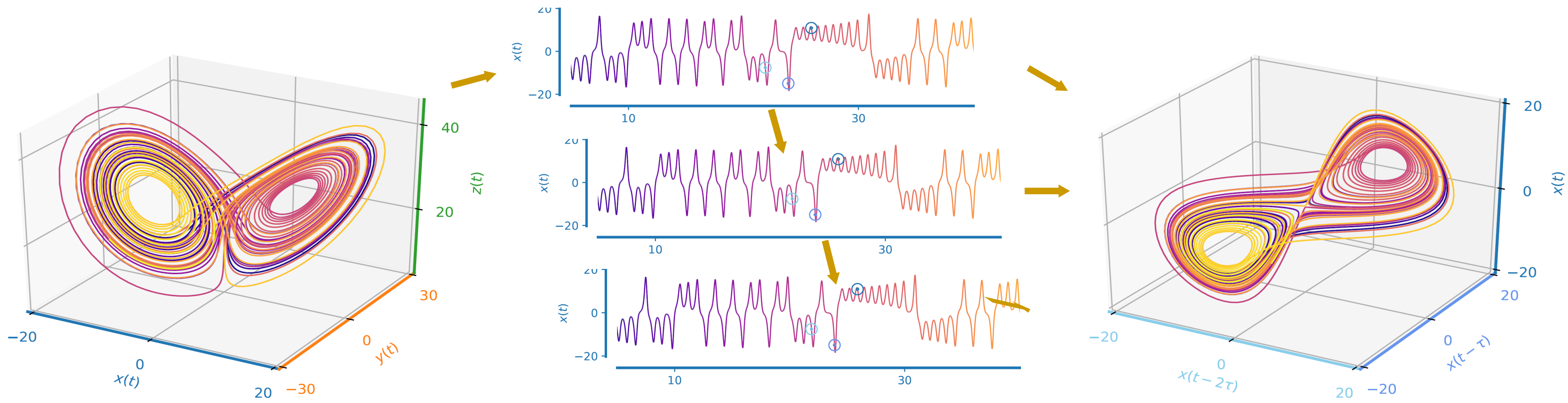
- Logistic map

$$x_{t+1} = r x_t (1 - x_t)$$





Theory of dynamical systems: Takens' embedding theorem



Let us observe a timeseries of a scalar variable: x(t)
That is a two times continouosly derivable function of the state variables. Lets generate a vector from the time delayed vaues of x:

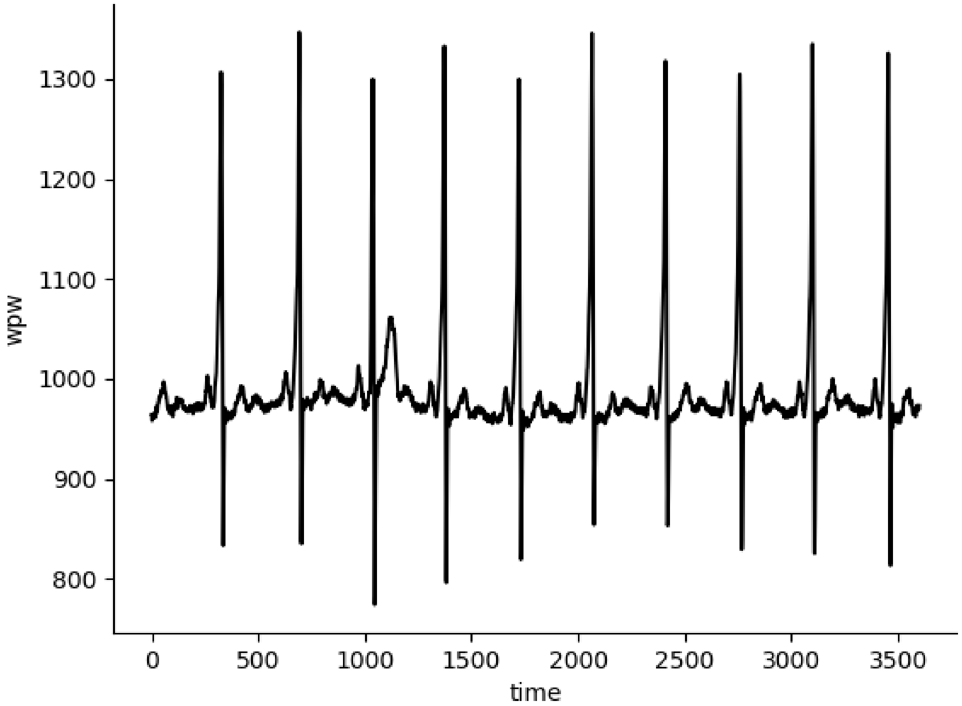
[x(t), x(t-τ), x(t-2τ), … x(t-nτ)] is an embedding, if n>2d+1

The pseudo-attractor of the system in this reconstructed state space is topologically equivalent with the system's real d dimensional attractor in its real state space.

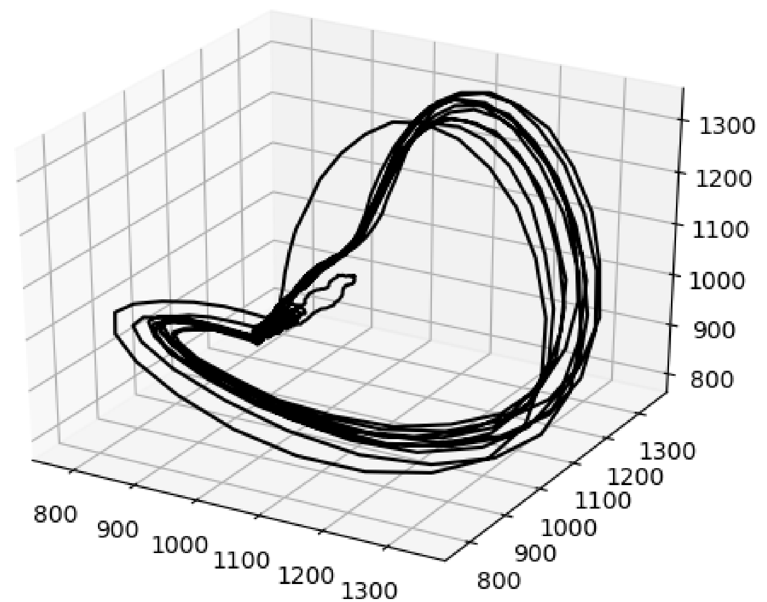
Theory of dynamical systems: Takens' embedding theorem

What to do with the reconstructed attractors?

Example: ECG from a patient with Wolff–Parkinson–White (WPW) syndrome:



3D Embedding



Phase space reconstruction

What to do with the reconstructed attractors?

It is not easy to determine the type (topology) of the attractor, based on the noisy measurements.

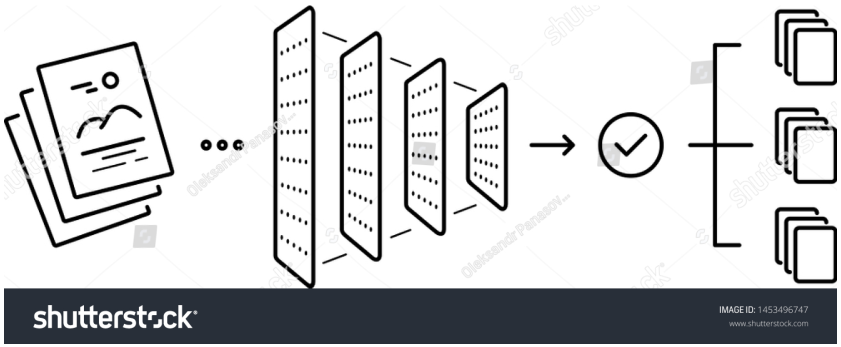
Dimension is a topological invariant, so it is possible to measure its dimension.

It is possible to measure the average Ljapunov-exponent, meaning the average instability of the trajectories.

What else?

Let’s detect anomalies!

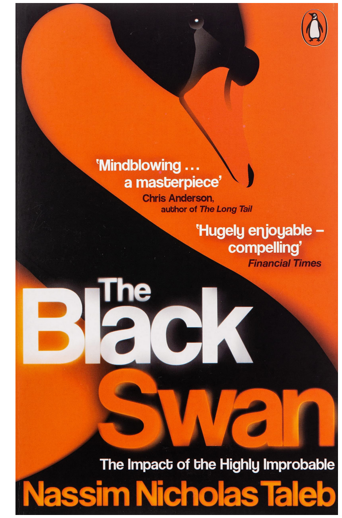
Supervised



Anomaly detection approaches

New type of anomalies: Unique events

- Not necessarily very different



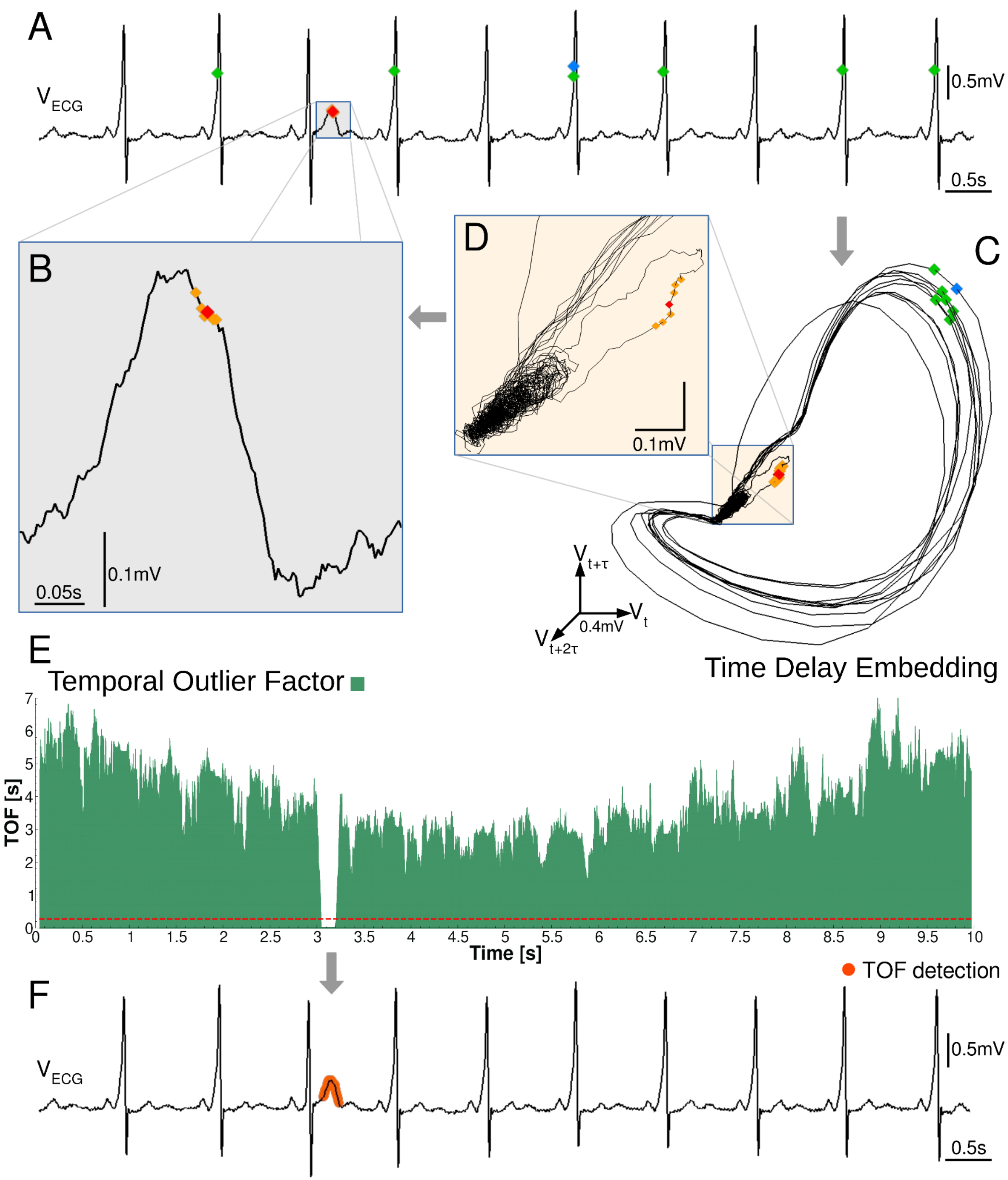
- But unique!
- How to define uniqueness in continuous variables?

How to find a unicorn?

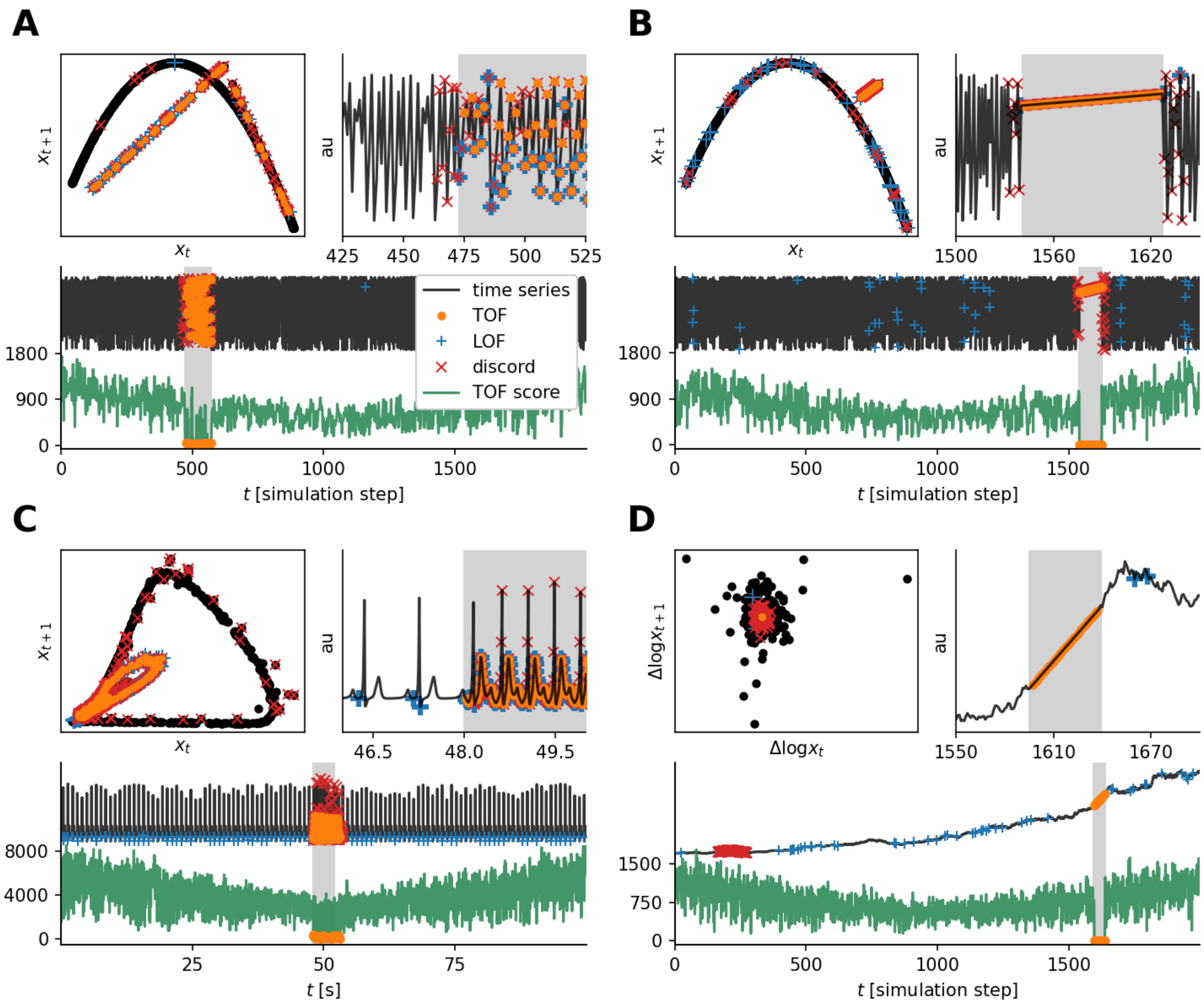
Unique and non-unique points points on continuous data series

The nearest neighbors of the state ◆ are the states ◆. They are evenly distributed in time, thus the system returned to this state time-to-time.

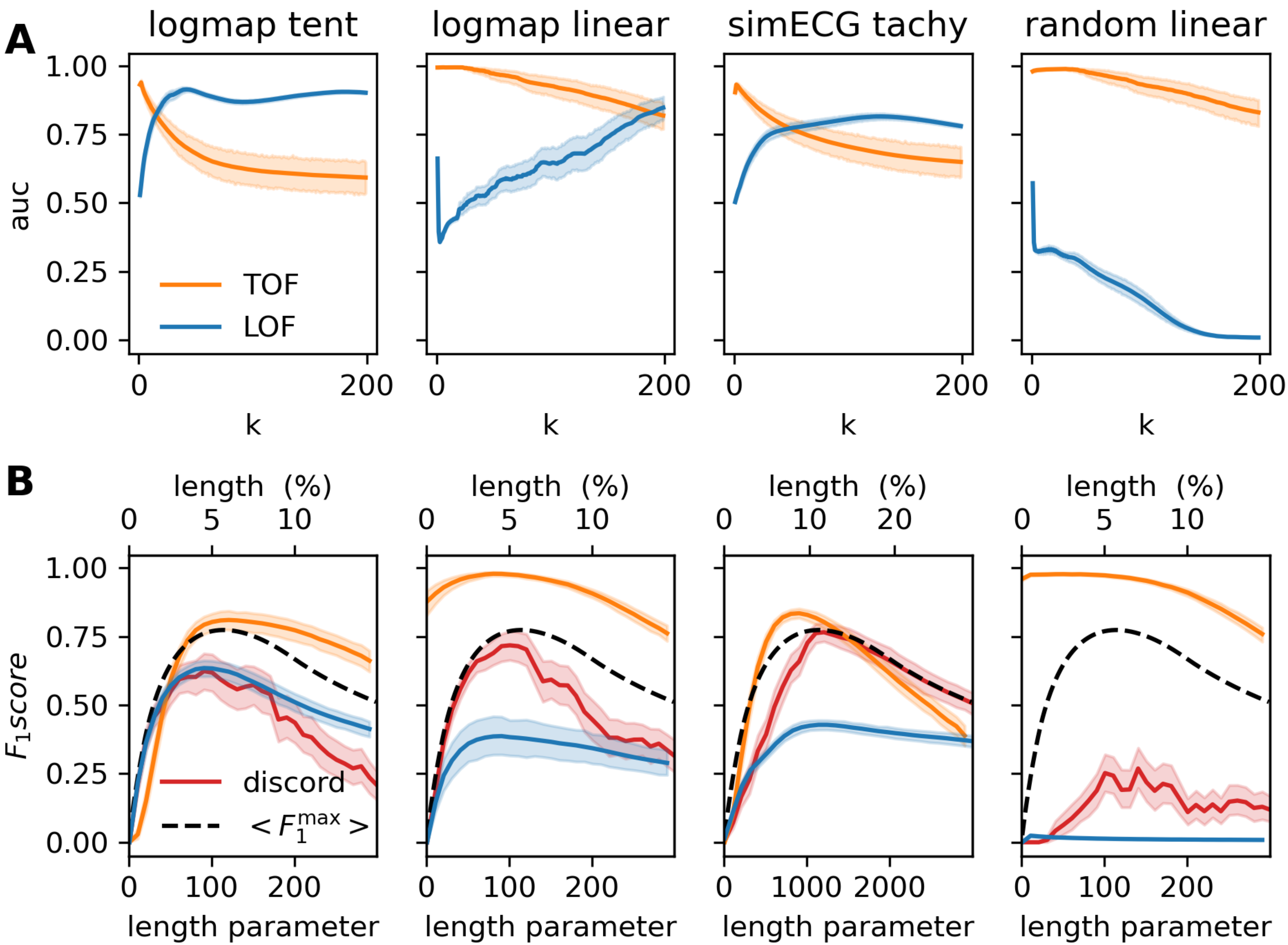
The nearest neighbors of the state ◆ are the states ◆ . All of them are neighbours of state ◆ in time as well. Thus the system never returned to this state: it is a unique state.



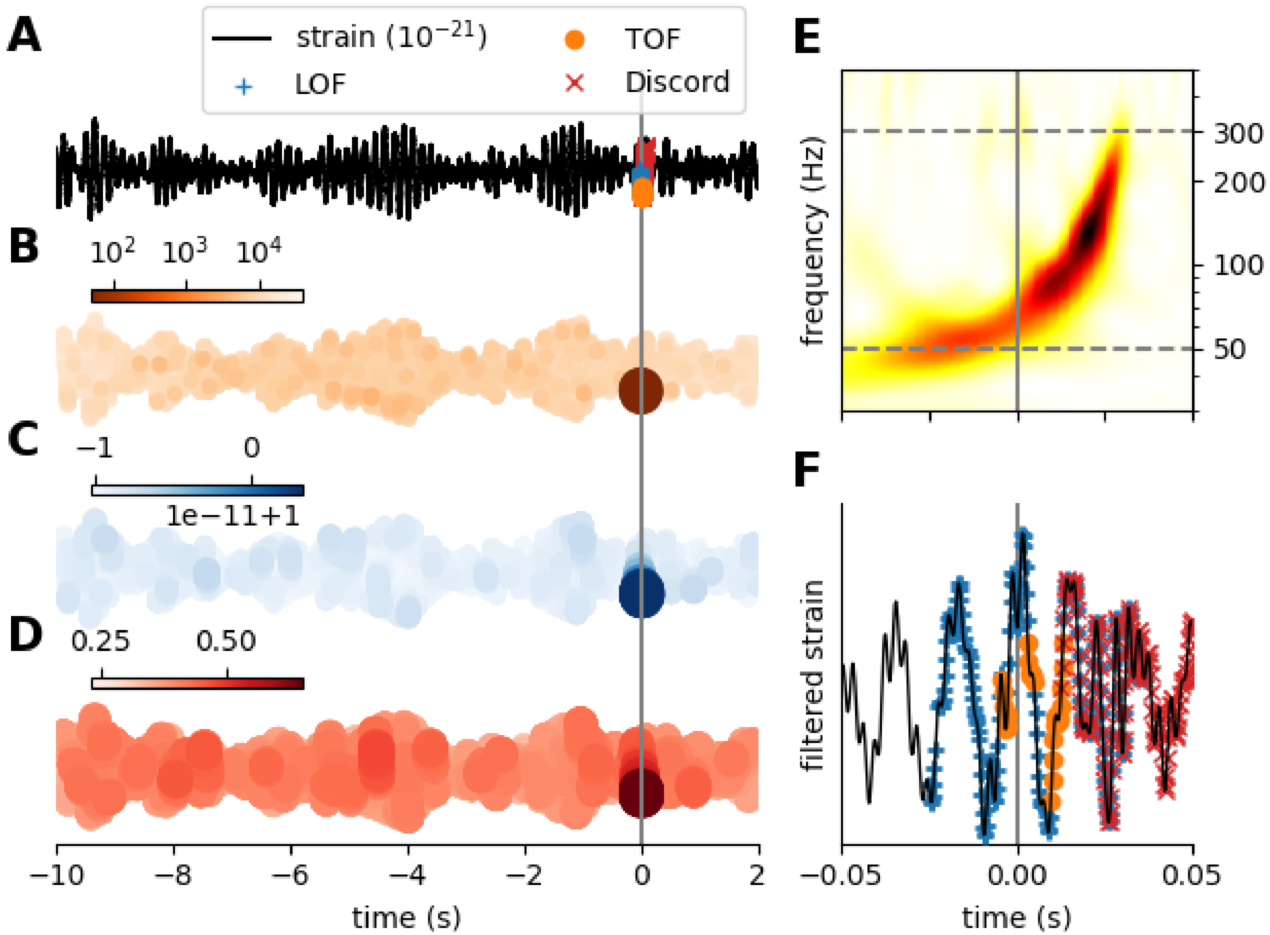
Tests on simulations

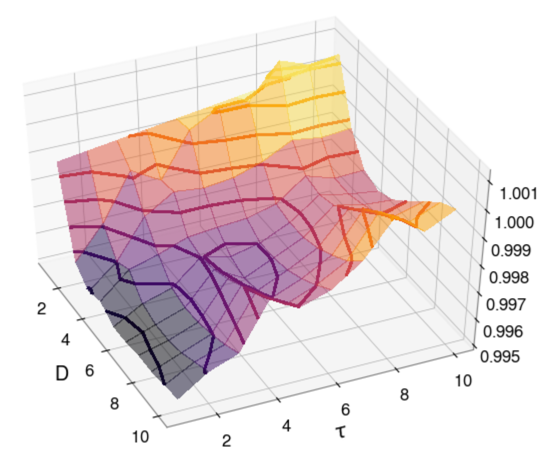


Tests on simulations



What is already known: GW 170817
Gravitational waves of the merger neutron stars



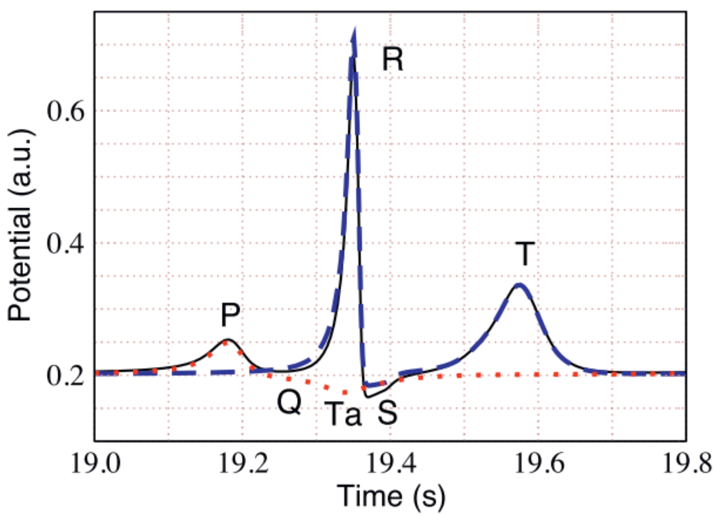


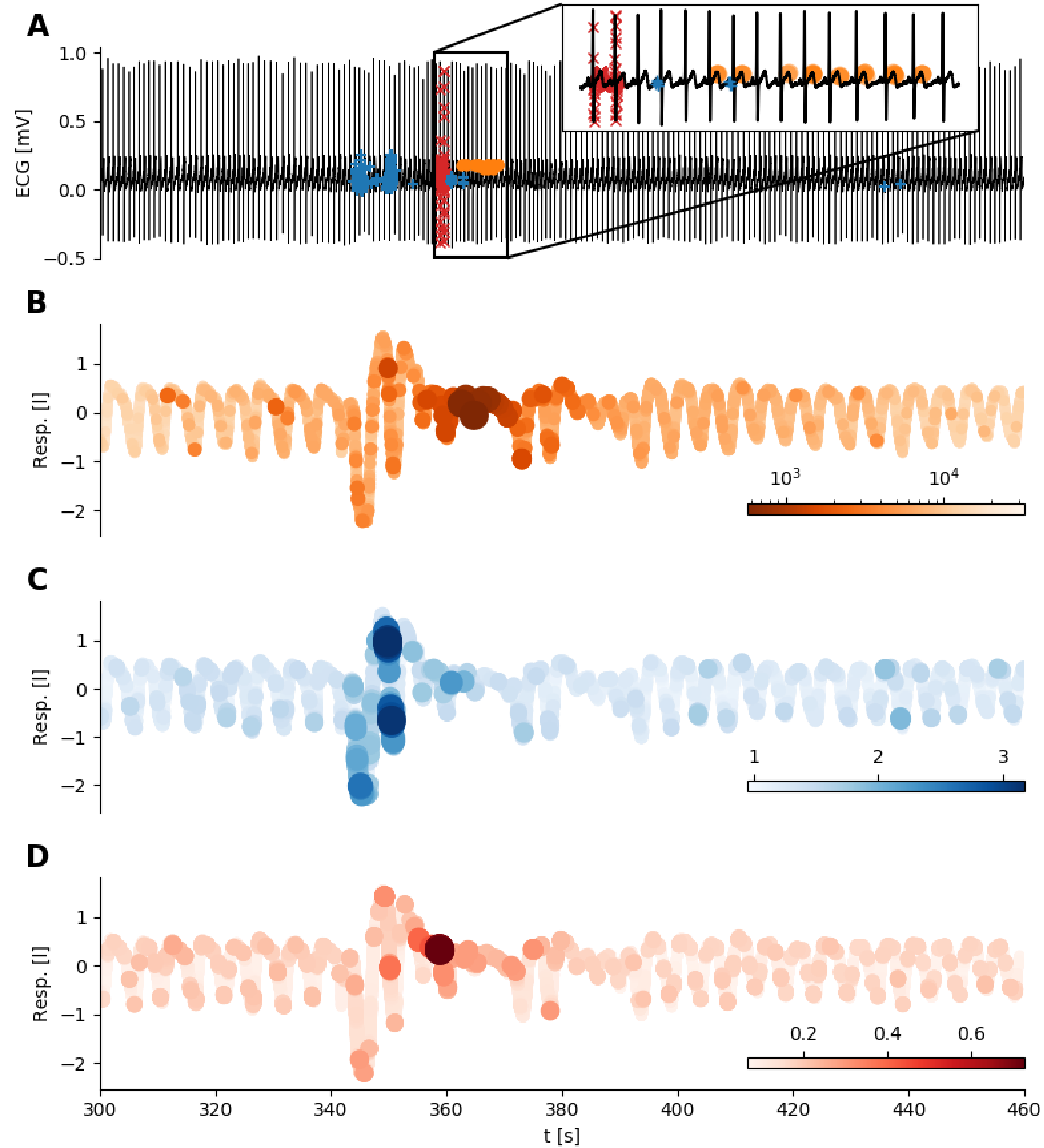
Abbott, B. P., Abbott, R., Abbott, T. D., Acernese, F., Ackley, K., Adams, C., … Woudt, P. A. (2017). Multi-messenger Observations of a Binary Neutron Star Merger. The Astrophysical Journal, 848(2), L12. https://doi.org/10.3847/2041-8213/aa91c9

Gautama, T., Mandic, D. P., & Hulle, M. M. Van. (2003). A differential entropy based method for determining the optimal embedding parameters of a signal. 2003 IEEE International Conference on Acoustics, Speech, and Signal Processing, 2003. Proceedings. (ICASSP '03)., 6, VI–29.

Signs of sleep anpoe on ECG data series

Interestingly the t-waves were unique, which are known to be useless in ECG diagnostics.

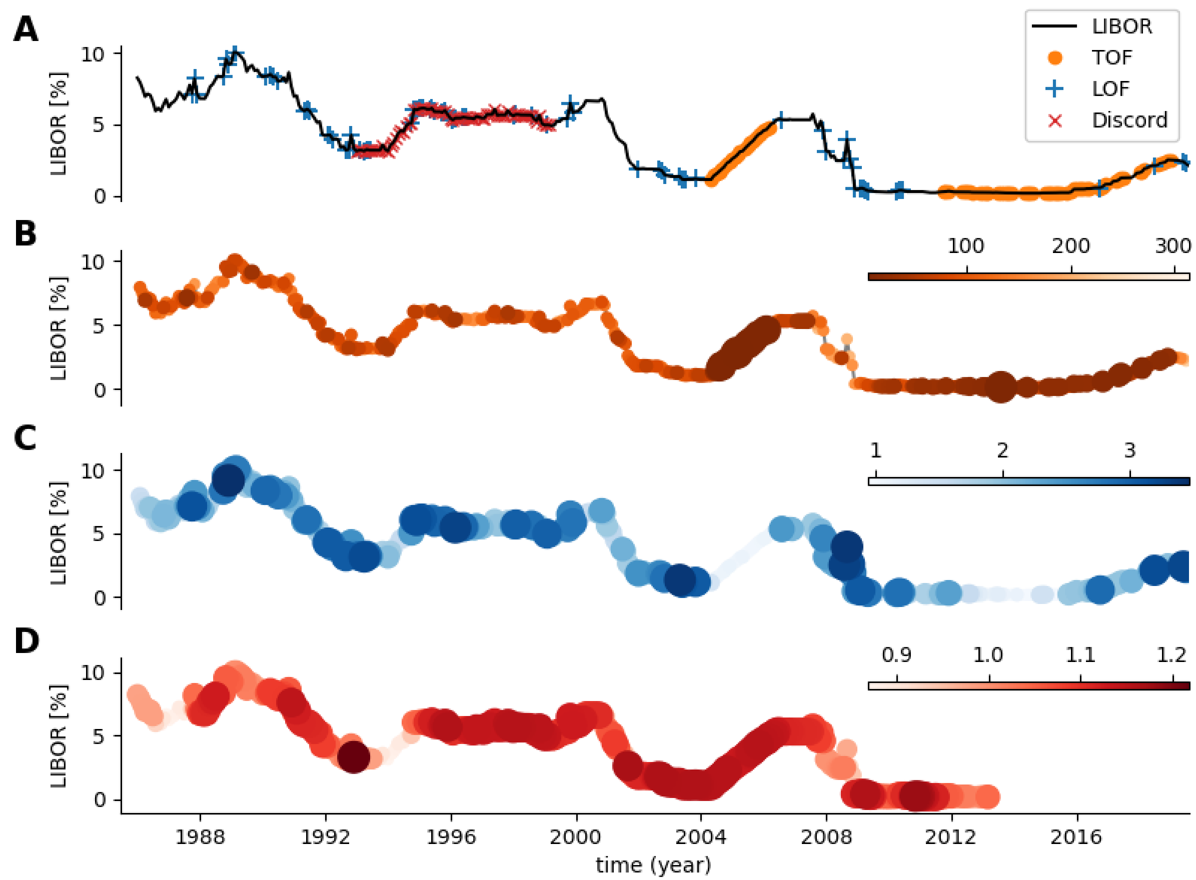




London interbank offered rate (LIBOR)

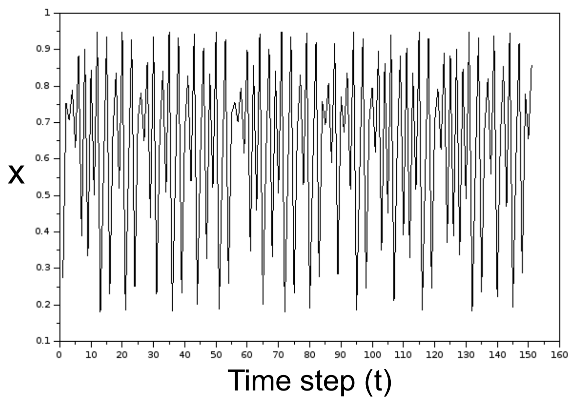
Between 2005 and 2008 there were serious manipulations by some of the most significant banks.

In 2012 the calculation system has been reorganized.

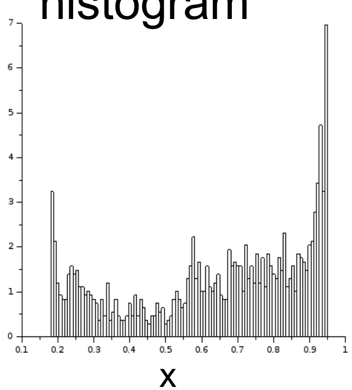


Chaos and Noise

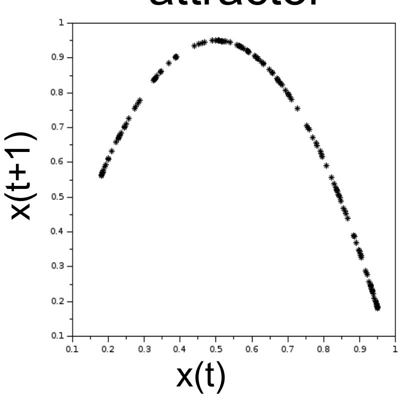
Chaotic time series – generated by a deterministic map: $x(t+1) = r\,x(t)(1-x(t))$



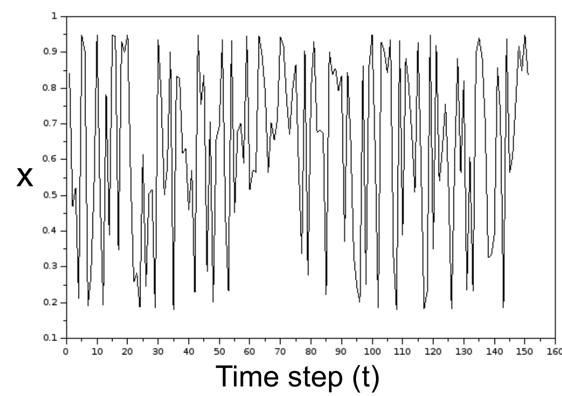
Amplitude histogram



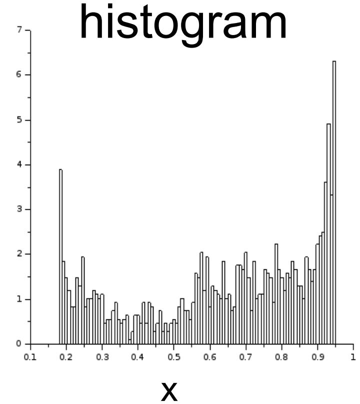
Low dimensional attractor



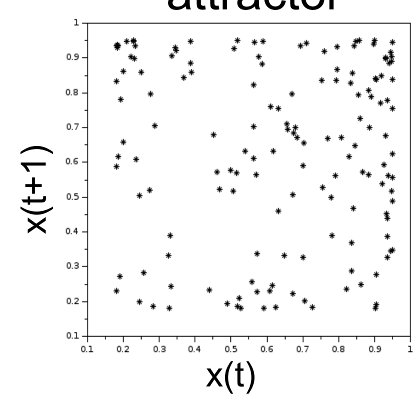
Stochastic (noise) time series – resulted by a random permutation of the chaotic time series



Amplitude histogram



High dimensional attractor



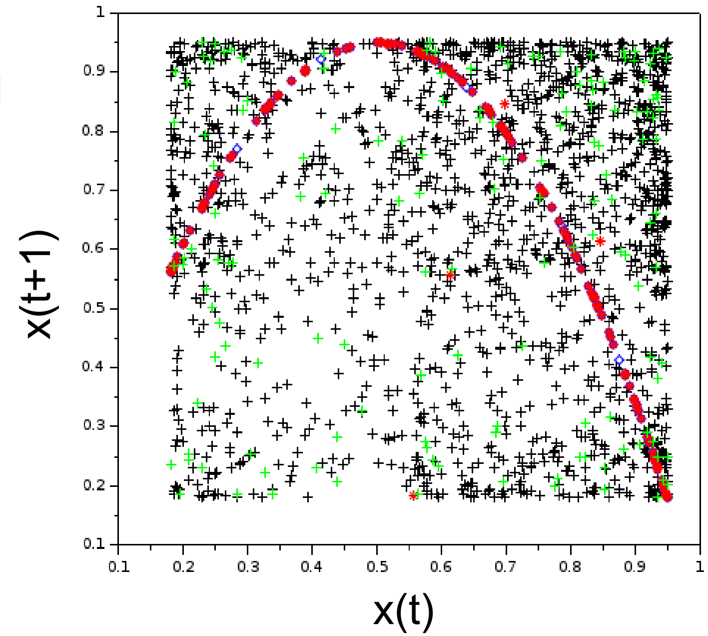
Chaotic anomalies in stochastic data series

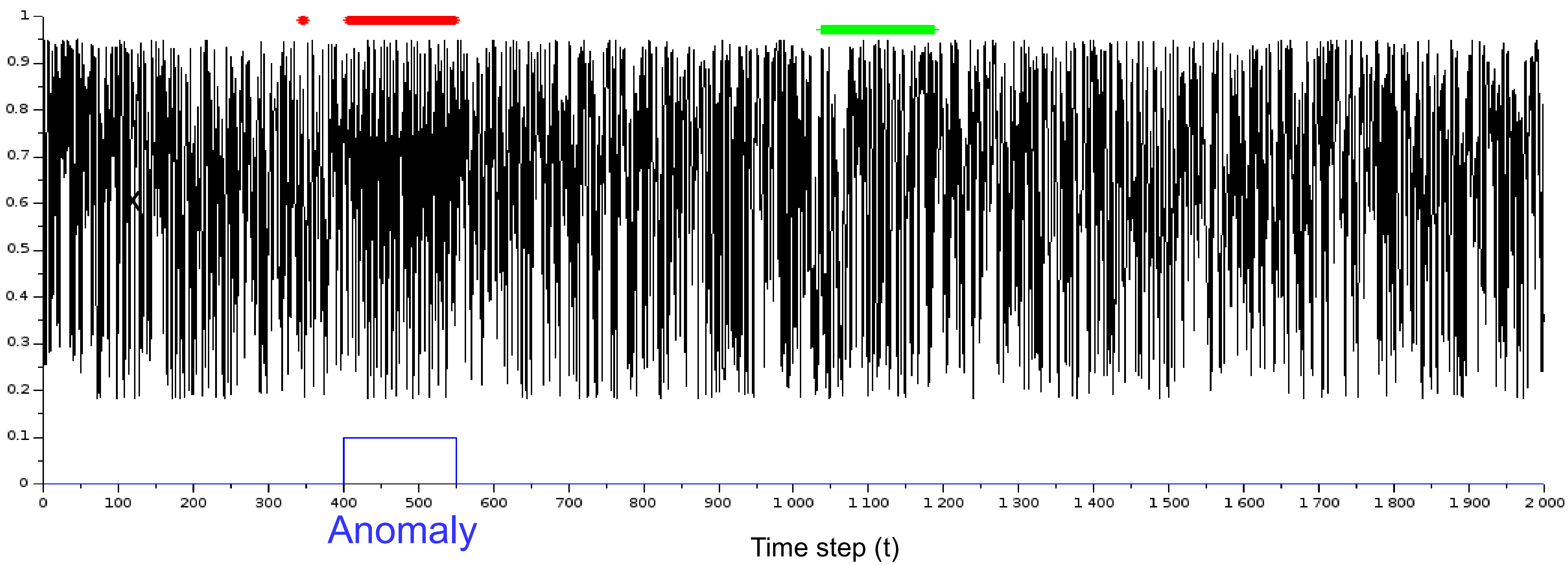
State space reconstruction – time delay embedding

Points belong to the chaotic anomaly

Detected unicorn

Discord detection





Noise anomalies in chaotic dataseries

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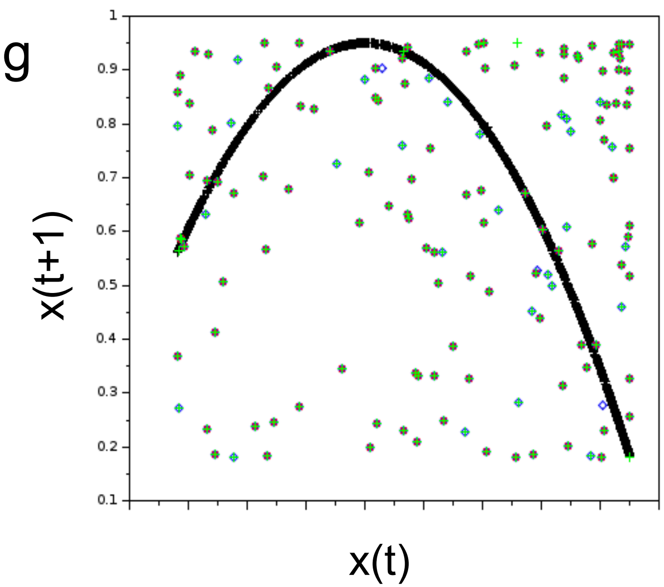
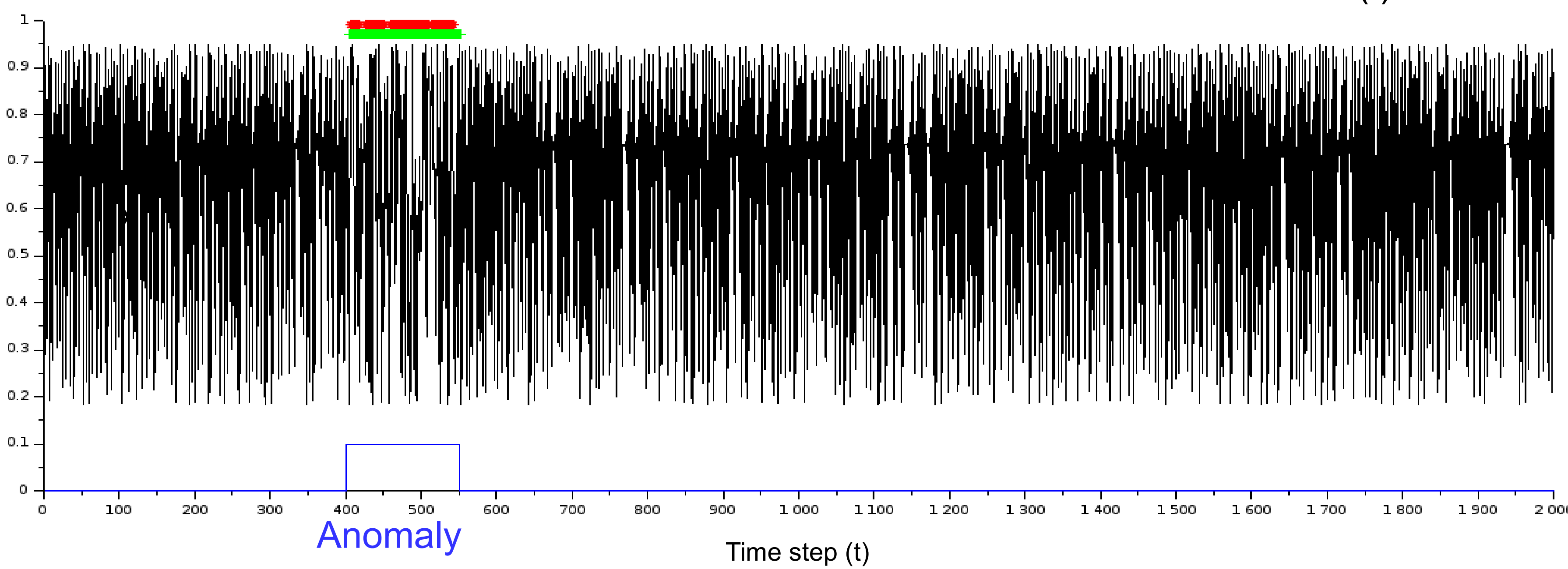
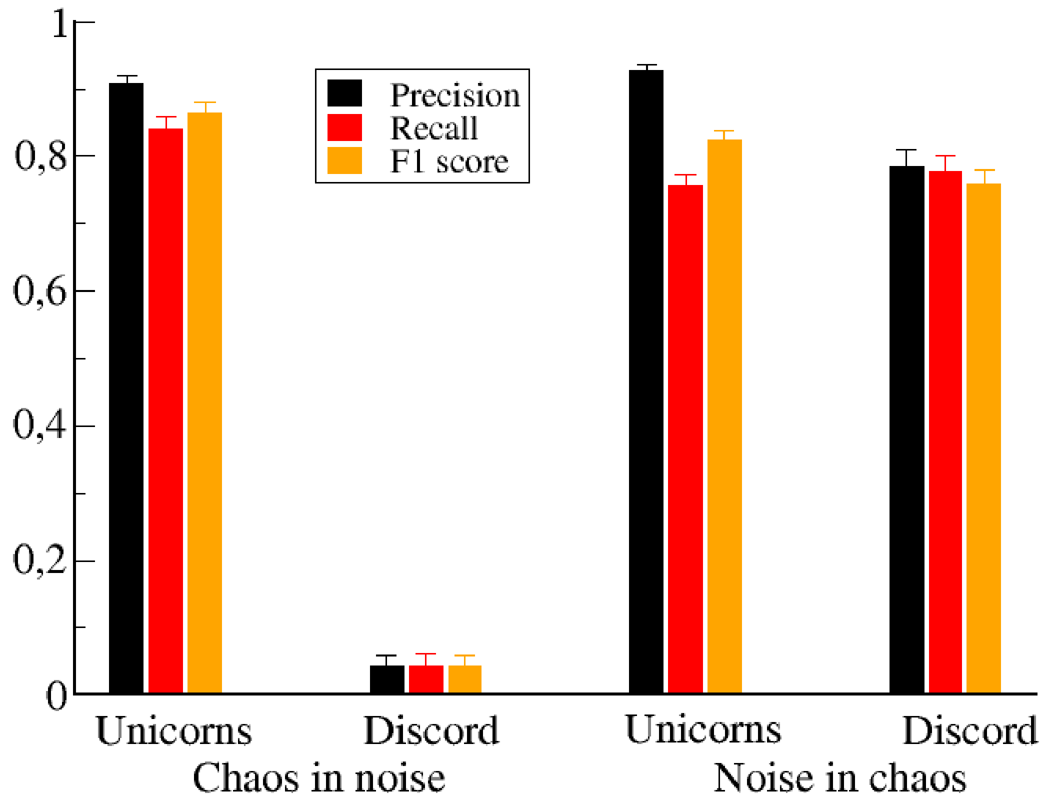




Noise in chaos and chaos in noise

The same algorithm was able to detect

- chaotic anomalies on stochastic background
- Noise anomalies on chaotic background



Determination of causal relationships between time series and applications to neural data

Zoltán Somogyvári

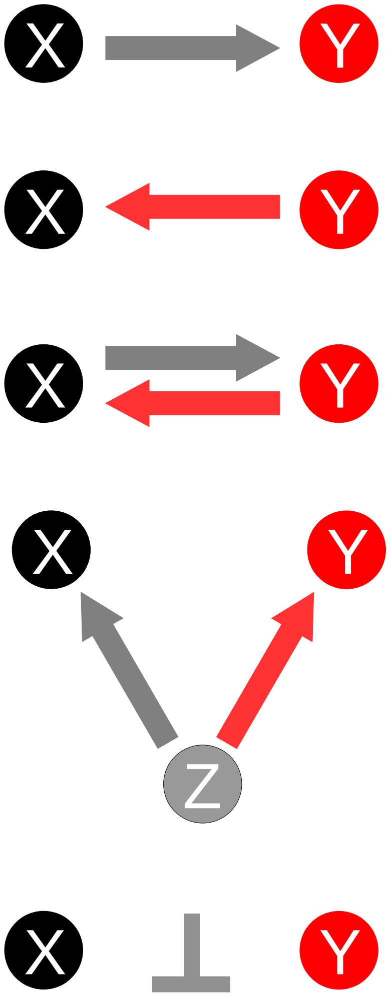
Research Centre for Physics
Department of Computational Sciences

Determination of causal effects in time series

Is there any possibility to identify directed causal relationships from two observed data series, without experimental intervention?

We surely can measure correlation, but correlation and causality are different things. Moreover correlation is an asymmetrical relation while causality can be unidirectional.

Is there a way to distinguish directional and bidirectional (circular) causality or to reveal hidden common cause?



Personalized medicine:
Causality analysis in epilepsy

Is there a (multiple) Seizure Onset Zone (SOZ) or rather an epileptic network?

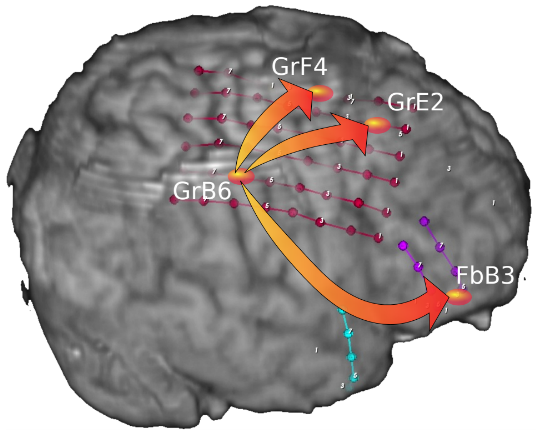
The SOZ is causal source during the seizure?

What about during interictal periods?

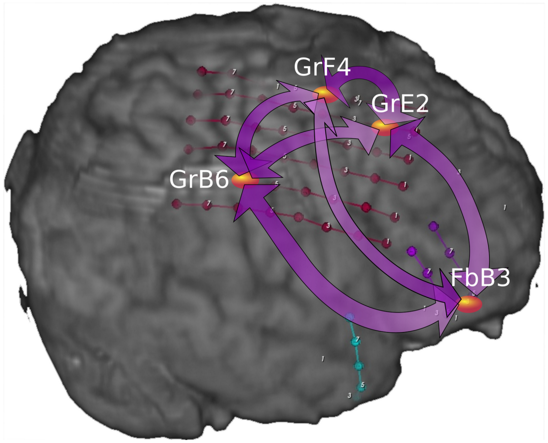
Sabesan, S., Good, L.B., Tsakalis, K.S., Spanias, A., Treiman, D.M., Iasemidis, L.D.: Information flow and application to epileptogenic focus localization from intracranial EEG. IEEE Trans Neural Syst Rehabil Eng 17(3), 244–253 (2009)

Epstein, C.M., Adhikari, B.M., Gross, R., Willie, J., Dhamala, M.: Application of high-frequency Granger causality to analysis of epileptic seizures and surgical decision making. Epilepsia 55(12), 2038–2047 (2014)

Seizure onset zone

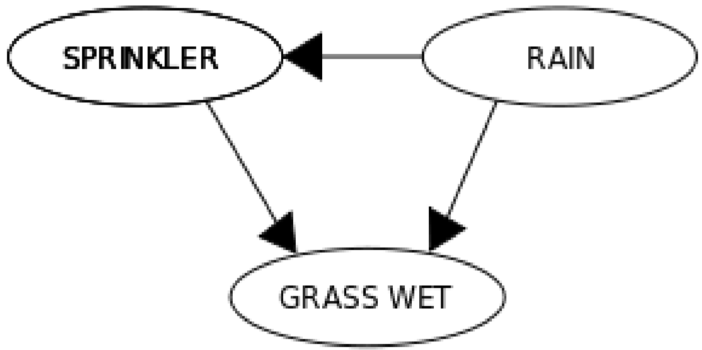


Epileptic network



With interventions: Bayesian networks, graphical models, Conditional independence

RAIN	SPRINKLER T	SPRINKLER F
F	0.4	0.6
T	0.01	0.99



RAIN T	RAIN F
0.2	0.8

SPRINKLER	RAIN	GRASS WET T	GRASS WET F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

Judea Pearl

The theory allows to reveal the direction of the dependencies only in specific cases or the direction of the relationships assumed a priory!

Bayesian networks with just observations

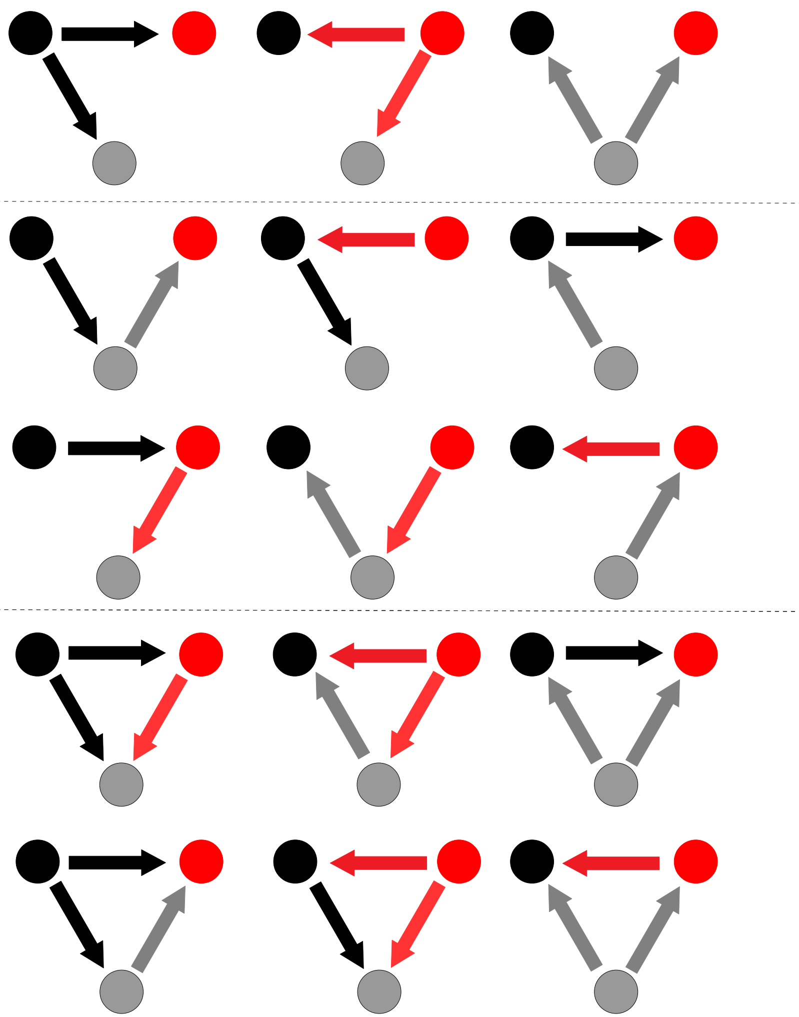
Observed dependencies

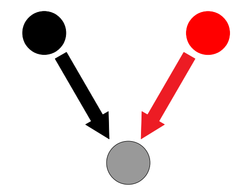
Truth

Forks

Chains

Combined





Collider

Can not infer direction for only two variables!

With interventions: Bayesian networks, graphical models, Conditional independence

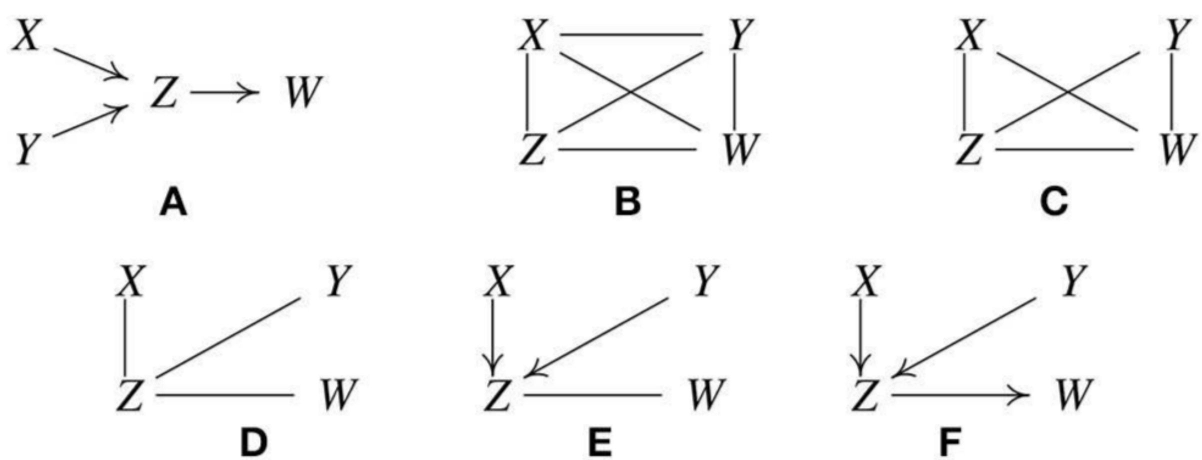


FIGURE 1 | Illustration of how the PC algorithm works. **(A)** Original true causal graph. **(B)** PC starts with a fully-connected undirected graph. **(C)** The $X - Y$ edge is removed because $X \perp\!\!\!\perp Y$. **(D)** The $X - W$ and $Y - W$ edges are removed because $X \perp\!\!\!\perp W \mid Z$ and $Y \perp\!\!\!\perp W \mid Z$. **(E)** After finding v-structures. **(F)** After orientation propagation.

Judea Pearl

In this specific case, the true relation structure can be determined, by using conditional independence tests only!

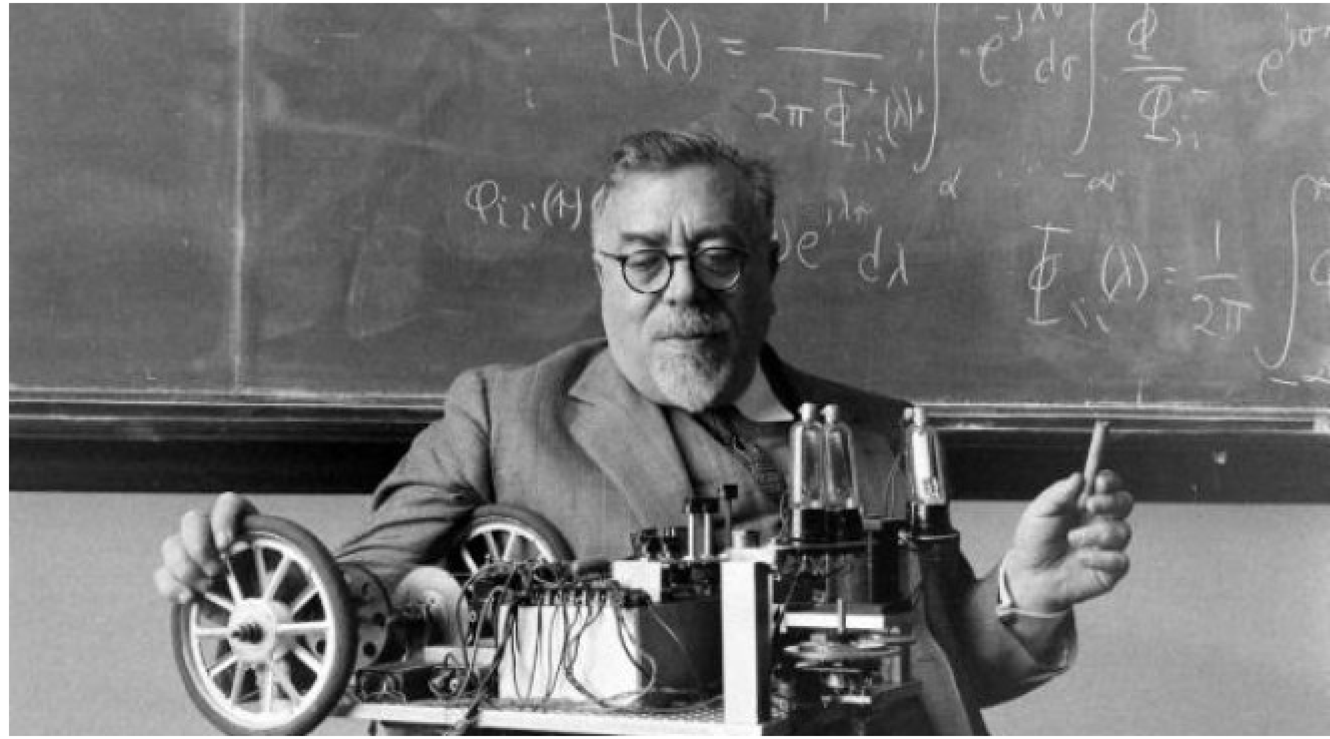
Times series: predictive causality

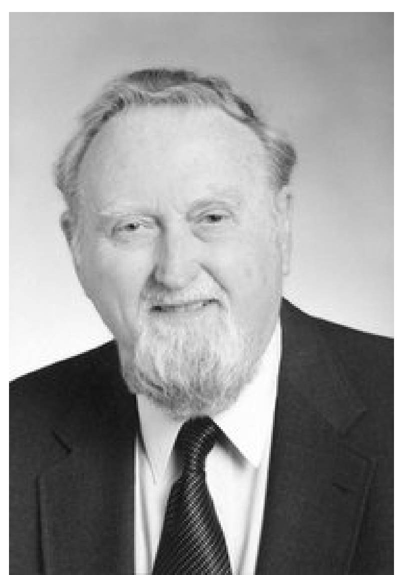
The original idea of predictive causality came from ***Norbert Wiener***

x → ***y***, if the inclusion of past ***x*** values improves the prediction quality on ***y***

X → Y

Assuming time delay via the concept of prediction helps to reveal direction!





Clive Granger implemented it via autoregressive linear models in 1969

Nobel price in Economic Sciences 2003

Granger- causality

Linear autoregression:

$$x_t = \sum_i a_i x_{t-i} + \varepsilon_1 \qquad x_t = \sum_i c_i x_{t-i} + \sum_i d_i y_{t-i} + \varepsilon_3$$

$$y_t = \sum_i b_i y_{t-i} + \varepsilon_2 \qquad y_t = \sum_i e_i x_{t-i} + \sum_i f_i y_{t-i} + \varepsilon_4$$

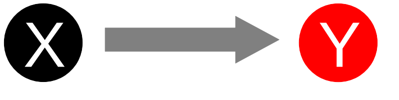
$$F_{y \to x} = \ln \left(\frac{var(\varepsilon_1)}{var(\varepsilon_3)} \right)$$

$$F_{x \to y} = \ln \left(\frac{var(\varepsilon_2)}{var(\varepsilon_4)} \right)$$

Evaluation F test:

$$F_{X \to Y} = \frac{\dfrac{var(\epsilon_1) - var(\epsilon_3)}{m}}{\dfrac{var(\epsilon_3)}{T - 2m - 1}}$$

It is sensitive to the model used for the prediction. The limitations of linear autoregressive models can be ameliorated by using nonlinear extensions, kernel solutions or model free transfer entropy method.



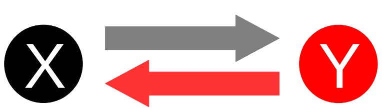
The model-free predictive causality: Transfer Entropy

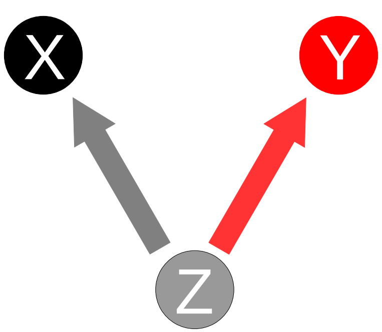
$$T_{X\to Y} = H(y_i|y^{(l)}_{i-t}) - H(y_i|y^{(l)}_{i-t}, x^{(k)}_{i-\tau})$$

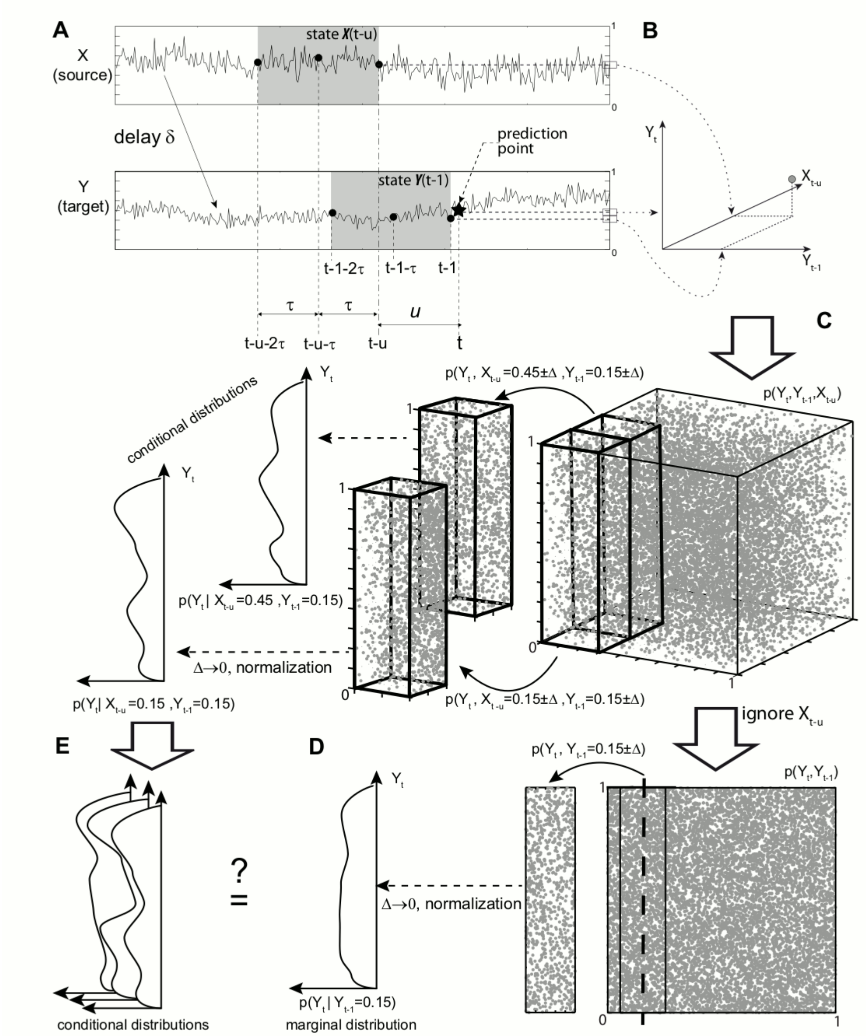
$$= \sum_{y_i, y^{(l)}_{i-t}} p(y_i, y^{(l)}_{i-t}) \log \frac{p(y^{(l)}_{i-t})}{p(y_i, y^{(l)}_{i-t})} - \sum_{y_i, y^{(l)}_{i-t}, x^{(k)}_{i-\tau}} p(y_i, y^{(l)}_{i-t}, x^{(k)}_{i-\tau}) \log \frac{p(x^{(k)}_{i-\tau}, y^{(l)}_{i-t})}{p(y_i, y^{(l)}_{i-t}, x^{(k)}_{i-\tau})}.$$

The framework of Judea Pearl (Bayesian nets) can not handle circular causal relationships.

Neither the Bayesian nets nor the predictive causality principle can not reveal the existence of unobserved hidden common causes between two variables







Cross Convergence Map:
A new framework for causality analysis

A new model-free approach, promising:

- Detection of circular causality
- Detection of nonlinear coupling

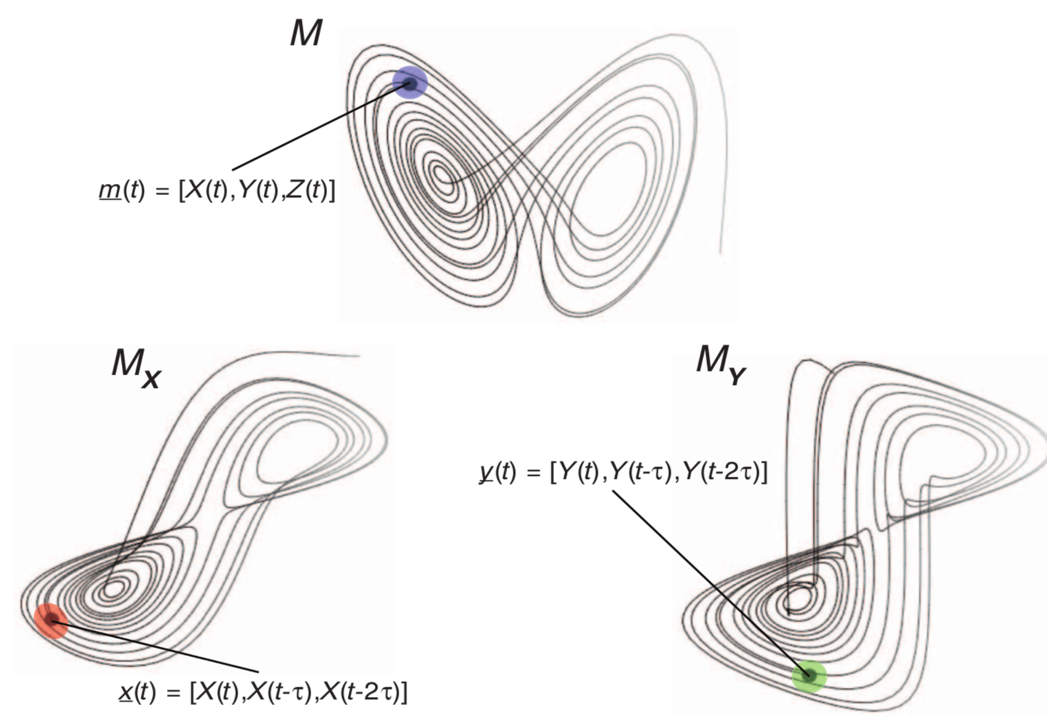
Detecting Causality in Complex Ecosystems

George Sugihara,[1*] Robert May,[2] Hao Ye,[1] Chih-hao Hsieh,[3*] Ethan Deyle,[1] Michael Fogarty,[4] Stephan Munch[5]

Science **338**, 496 (2012)

It utilizes the Taken's time delay embedding theorem:

The trajectory reconstructed in the state space is topologically equivalent With the trajectory of the system's original trajectory in its real space.



Cross Convergence Map:
A new framework for causality analysis

- Sugihara's method is based on that the consequence is an observation of the cause, thus the cause can be reconstructed from the consequence.
- Points that are neighbors in the state-space of the consequence should be neighbors in the state space of the cause as well.
- This topology preserving property can be tested by the cross mapping method.

Detecting Causality in Complex Ecosystems

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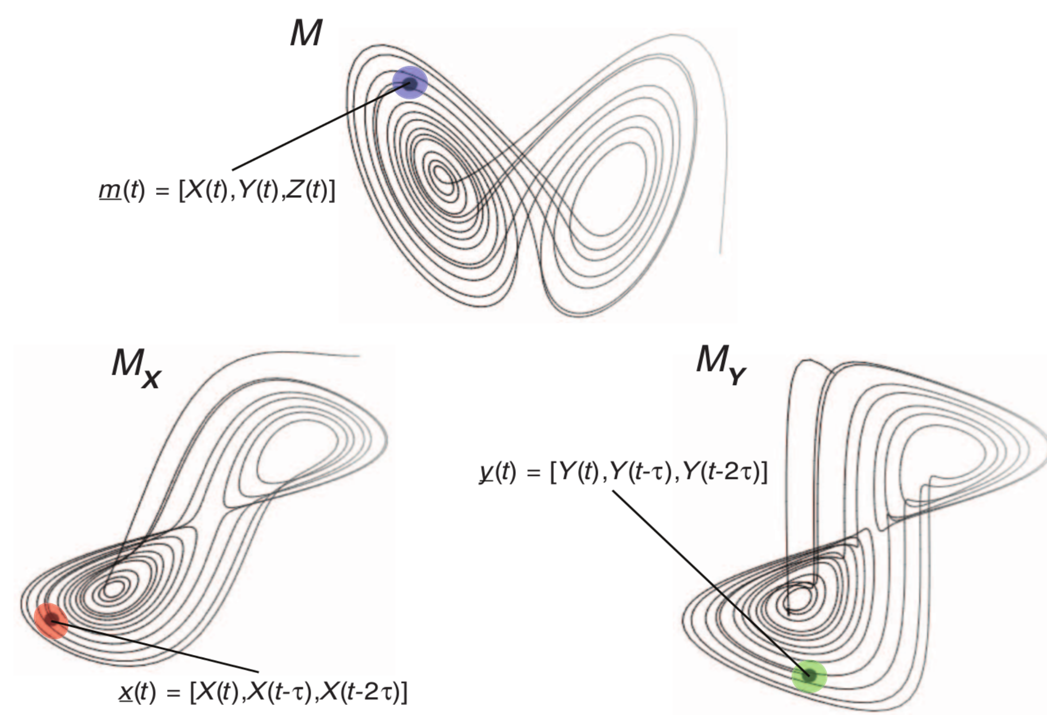


Nobel prize in Ecomomy 2021

In 2021, the work carried out in the field of causal relationships determined on the basis of observations again deserved the Nobel Prize in Economics:

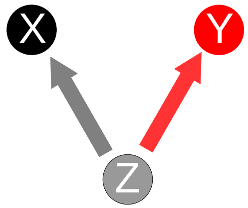
By studying accidental or one-time spontaneous phenomena, called as "natural experiments", the awardees defined social relationships that were previously only general beliefs in the absence of experimental data:

- Correlation between years spent studying and later earnings
- The minimum wage and unemployment
- Between immigration and changes in earnings.

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2021

Ill. Niklas Elmehed © Nobel Prize Outreach.	Ill. Niklas Elmehed © Nobel Prize Outreach.	Ill. Niklas Elmehed © Nobel Prize Outreach.
David Card	Joshua D. Angrist	Guido W. Imbens
Prize share: 1/2	Prize share: 1/4	Prize share: 1/4

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2021 was divided, one half awarded to David Card "for his empirical contributions to labour economics", the other half jointly to Joshua D. Angrist and Guido W. Imbens "for their methodological contributions to the analysis of causal relationships."

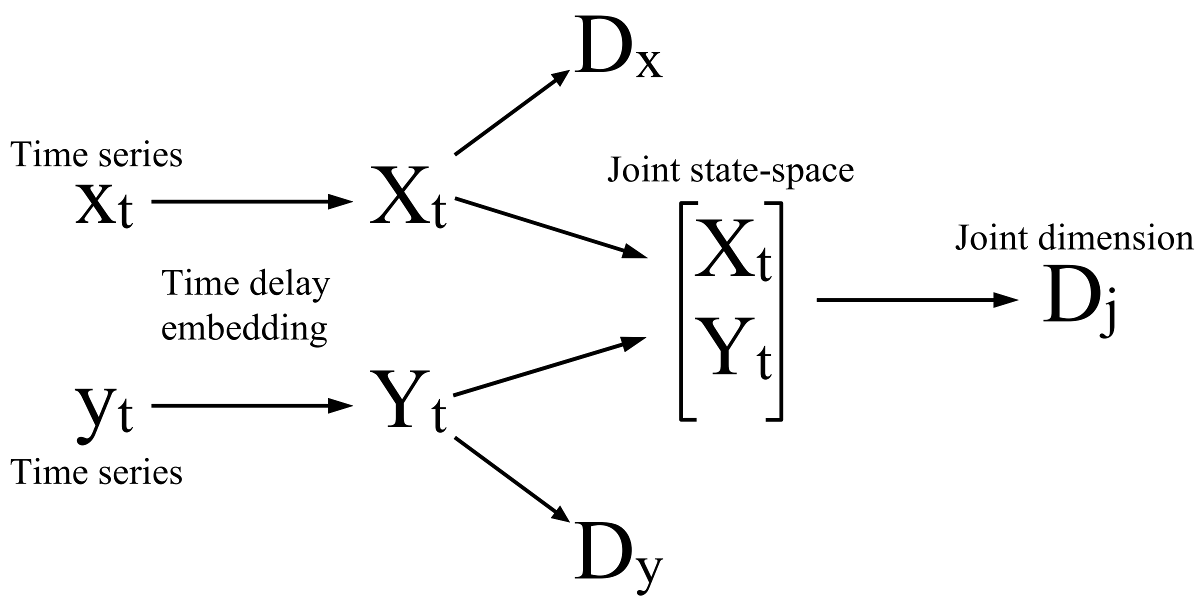


Revealing hidden common cause

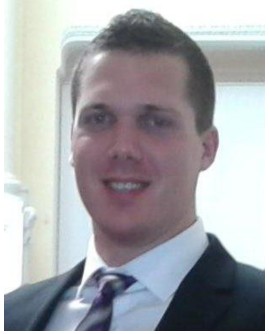
Neither Granger's nor Sugihara's method is able to detect the existence of a hidden common cause or distinguish it from the direct interaction.

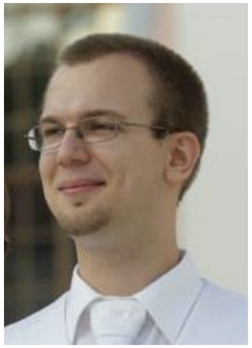
We have developed a new method which can!

It is based on the joint dimension measure:




Zsigmond Benkő


Ádám Zlatniczky


Marcell Stippinger

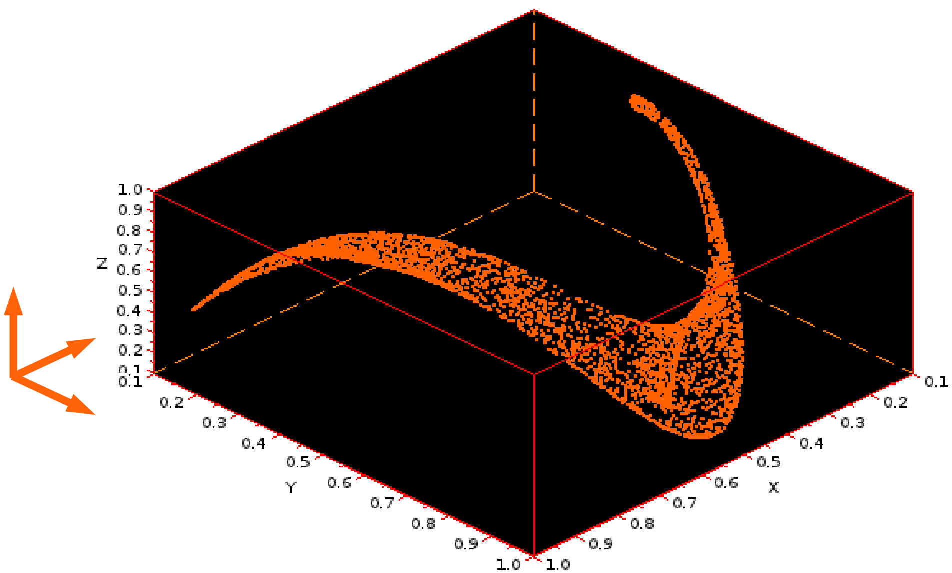

András Telcs

Revealing hidden common cause

Key point: the cause does not increases the dimension of the consequence in the joint space, the information is already there!

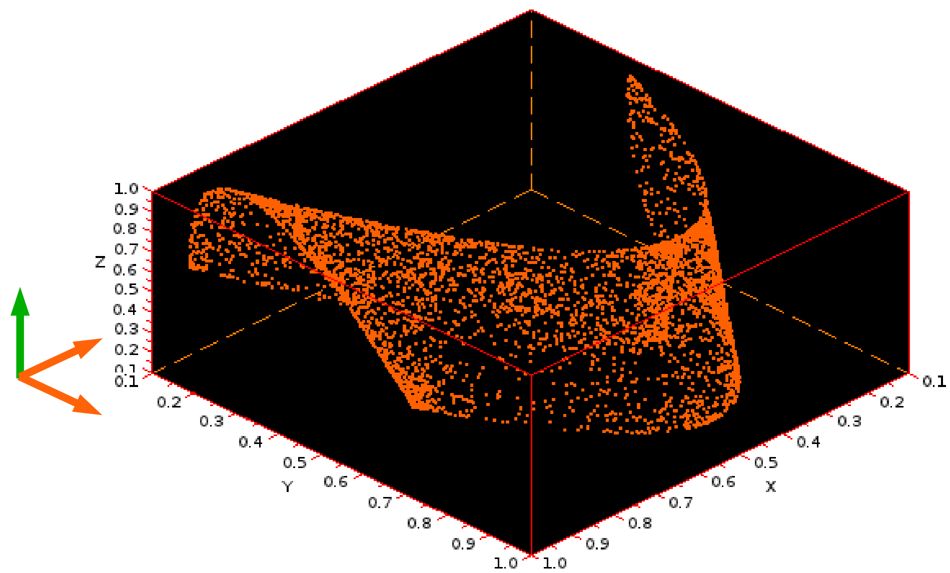
$x_{n+1}=r_x x_n((1-x_n)+b_{yx} y_n)$

$y_{n+1}=r_y y_n(1-y_n)$



$[x_n;x_{n+1};x_{n+2}]$

The consequence



$[x_n;x_{n+1};y_n]$

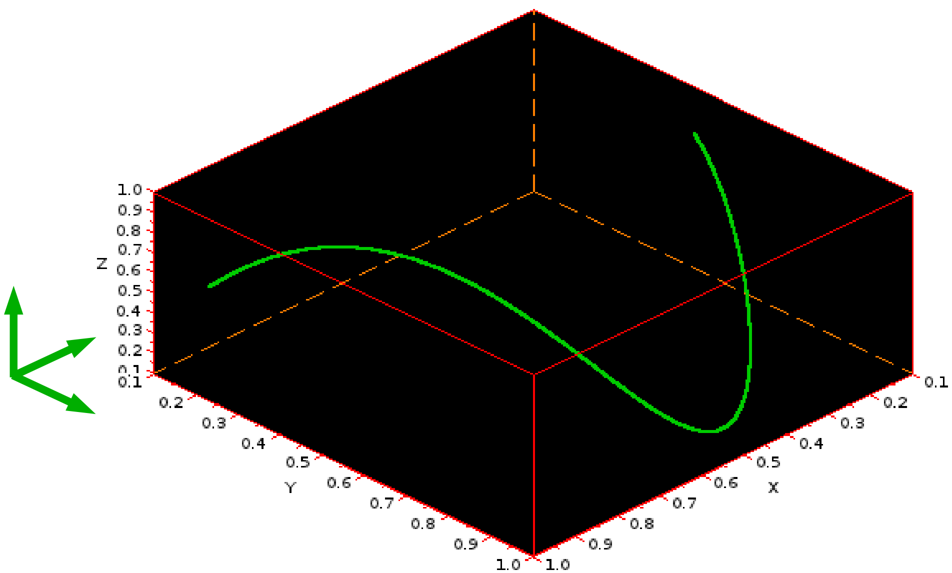
The cause and the consequence together in the joint space

The consequence formed a 2D manifold both in its own and the together with the cause in the joint state space. The lack of dimensionality increase in the joint dimension is the sign of the existing causal link (**x** depends on **y**).

Revealing hidden common cause

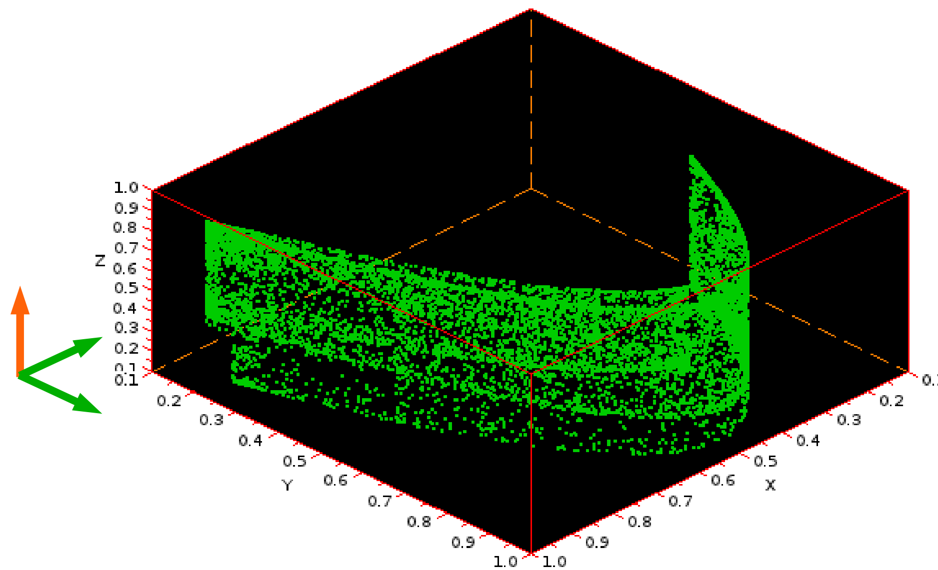
$x_{n+1}=r_x x_n((1-x_n)+b_{yx}y_n)$

$y_{n+1}=r_y y_n(1-y_n)$



$[y_n;y_{n+1};y_{n+2}]$

The cause

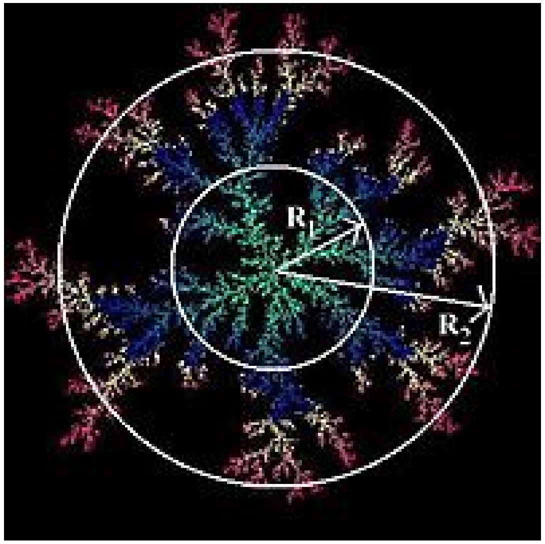


$[y_n;y_{n+1};x_n]$

The cause and the consequence together in the joint space

The cause formed a 1D manifold in its own, but a 2D manifold together with the consequence in the joint state space. The dimensionality increase in the joint state space is the sign of the independence (**x** contains different information compared to **y**, thus **x** does not cause **y**).

How to measure the dimension of the manifold?

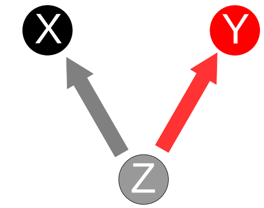


$$N(r) = N_0 \cdot r^D$$

$$D = \frac{Ln\left(\frac{N_i}{N_{i+1}}\right)}{Ln\left(\frac{r_i}{r_{i+1}}\right)}$$

Let's take two radii and count the number of points within the spheres: the exponent of the increase with respect to the radius gives us the dimension.

Revealing hidden common cause



Causal cases and the relations between the single and the joint dimensions:

Independence: $x_t \perp y_t \Rightarrow D_j = D_x + D_y$

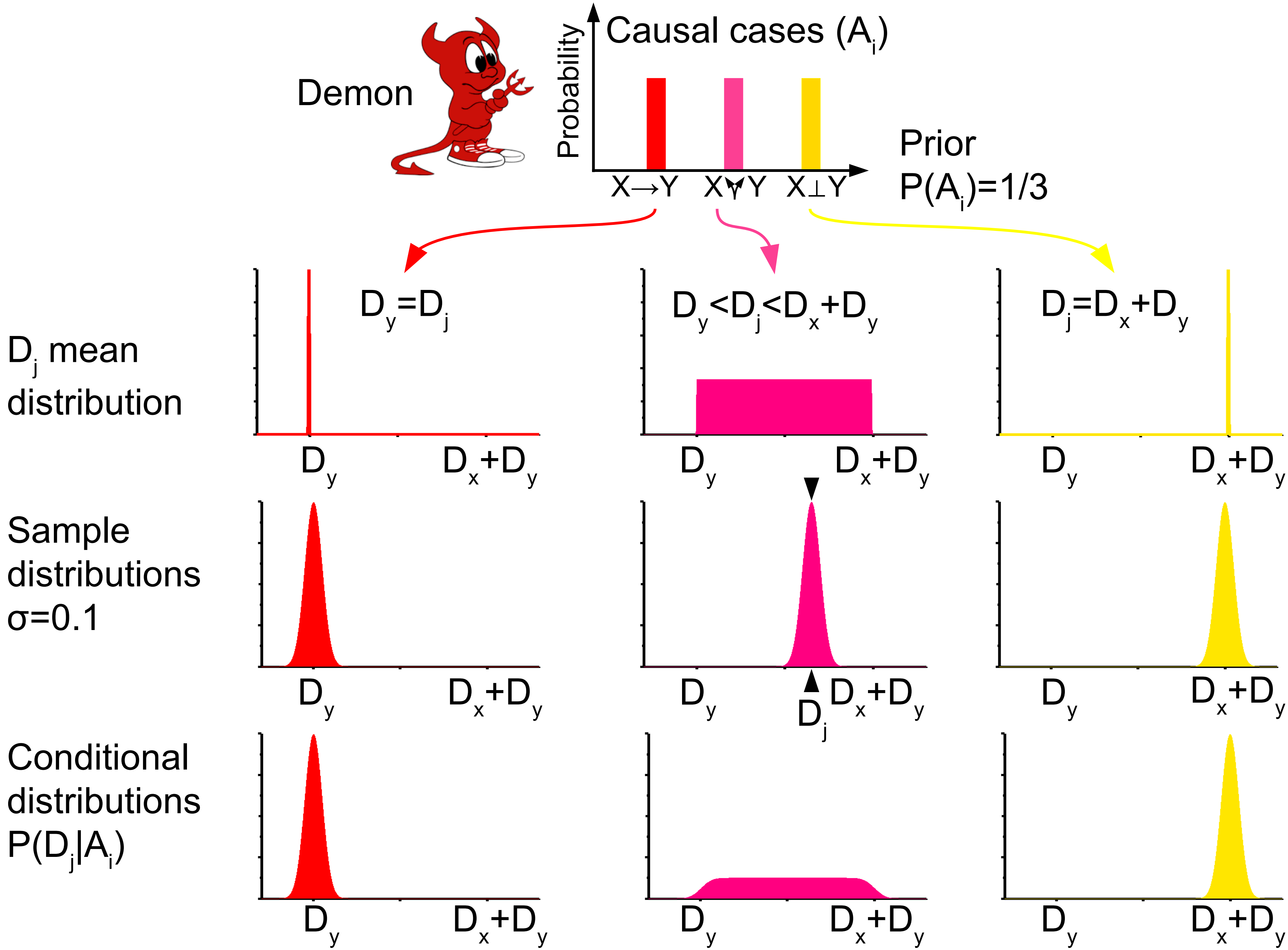
Unidirectional causality: $x_t \rightarrow y_t \Rightarrow D_j = D_y < D_x + D_y$

Circular causality: $x_t \leftrightarrow y_t \Rightarrow D_j = D_x = D_y$

Common cause: $x_t \curlyvee y_t \Rightarrow Max(D_x, D_y) < D_j < D_x + D_y$

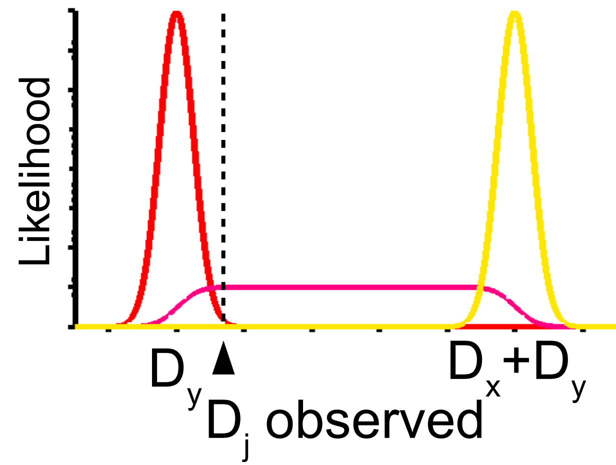
The type of the causal connection can be revealed by measuring the relations between the joint and the individual dimensions.

Bayesian model: a simplified version

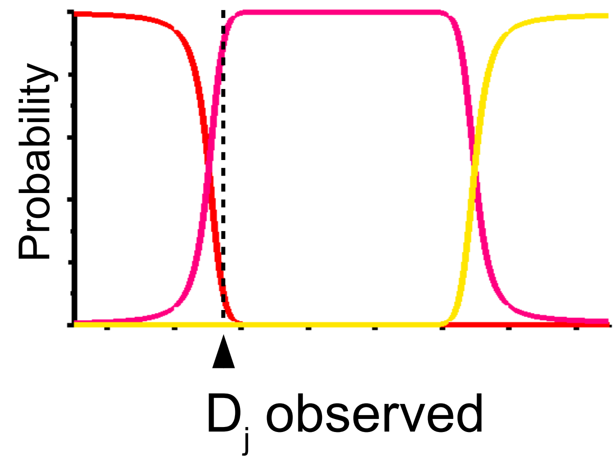


Bayesian inference: a simplified version

Likelihood
$P(D_j|A_i)$

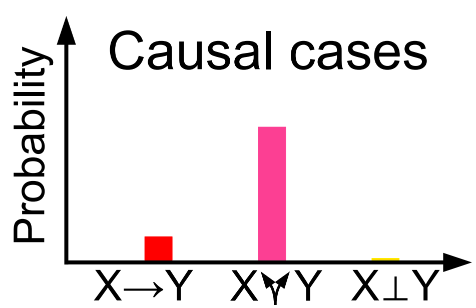


Conditional
posterior
distribution
$P(A_i|D_j)$



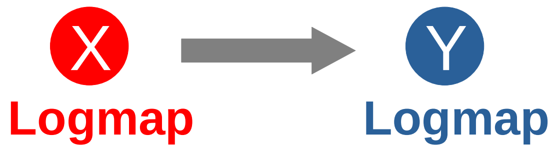
Bayes-theorem

$$P(A_i|D_j)=\frac{P(D_j|A_i)\,P(A_i)}{\sum P(D_j|A_i)\,P(A_i)}$$



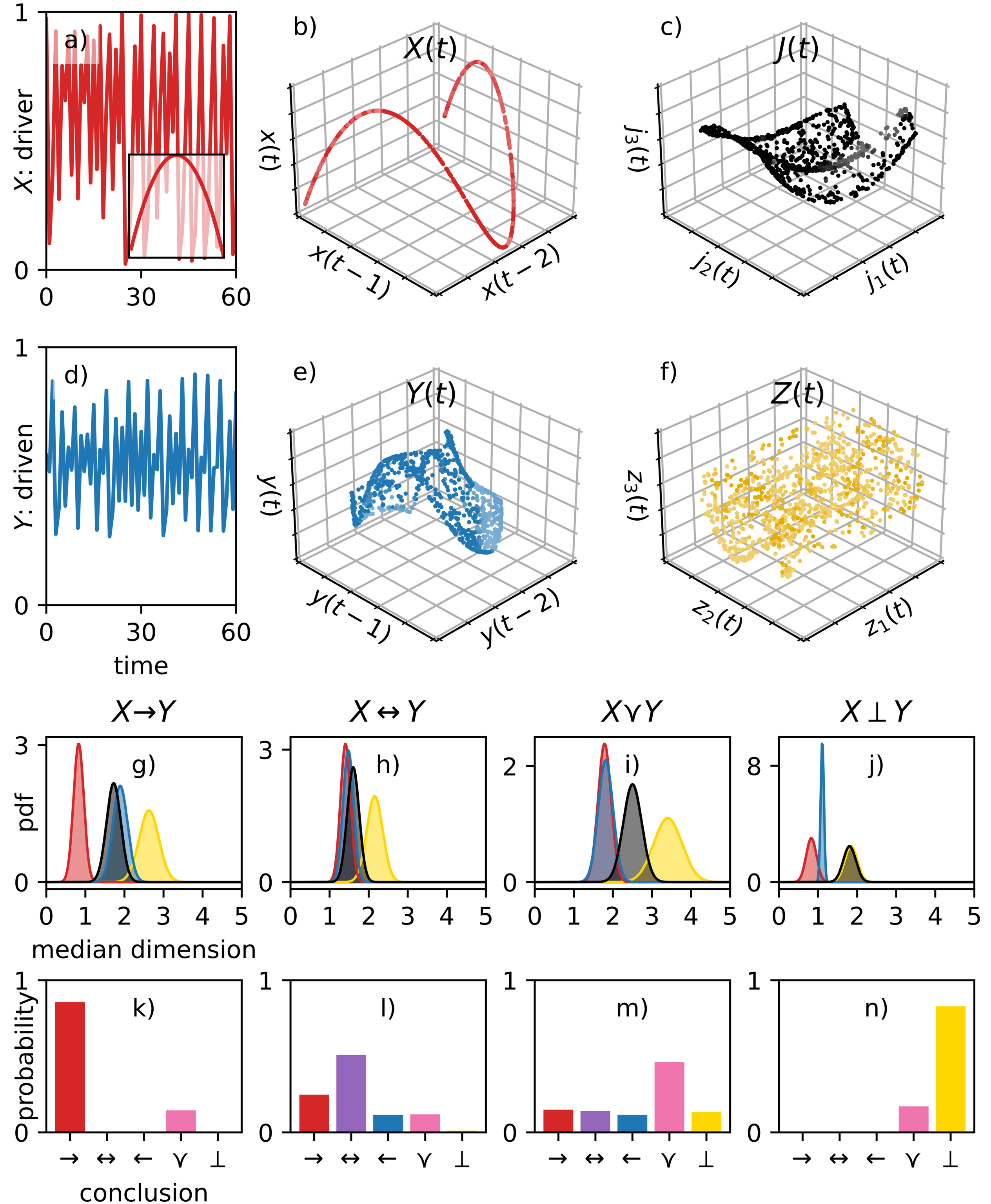
The workflow

Example:



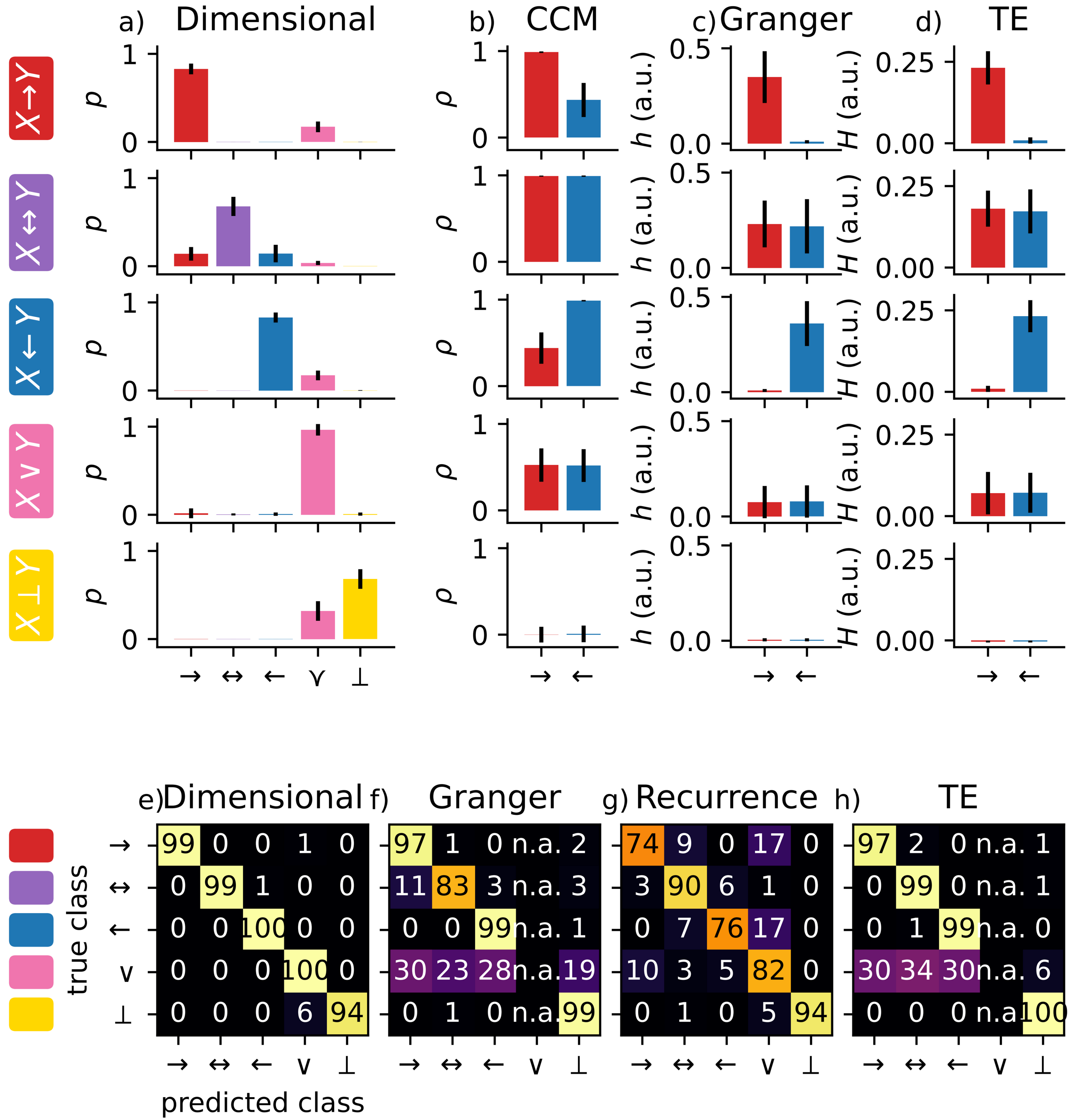
General:

1, Time delay embedding
2, Joint state space
3, Joint shuffled state space
4, Dimension measurements
5, Likelihood calculations
6, Probability calculations



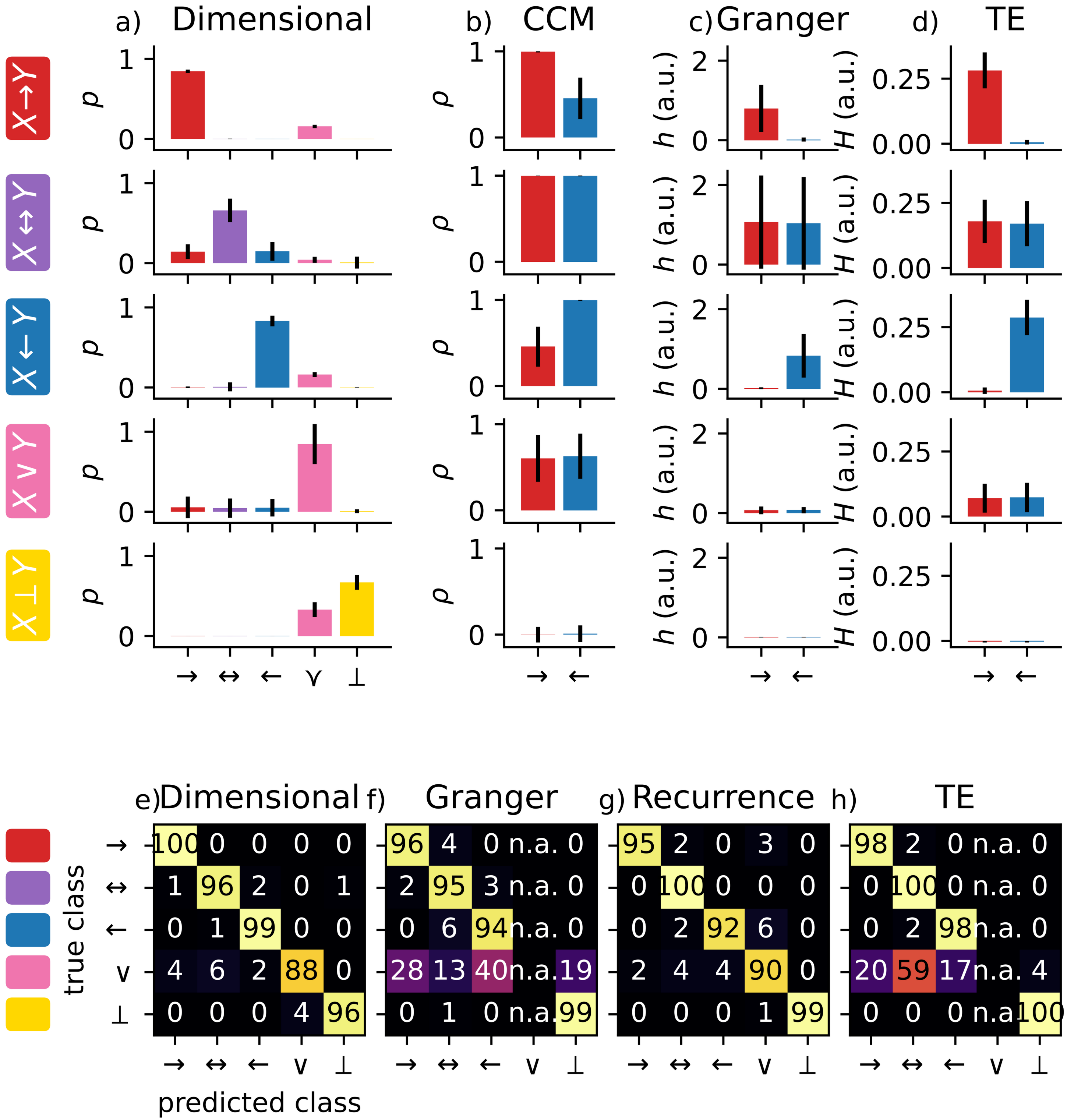
Test I. Coupled logistic maps

Logistic maps coupled in all possible cases. Here nonlinear couplings were used and the performance of four previous methods were compared.

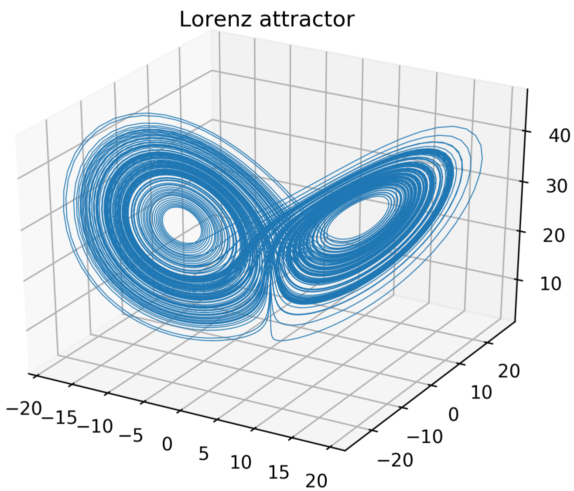


Test I. Coupled logistic maps

Logistic maps coupled in all possible cases. Here additive couplings were used and the performance of four previous methods were compared.



Test II. Coupled Lorentz systems



- 3 Lorenz systems: *X*, *Y*, *C*
- Each subsystem has 3 coordinates
- They are related through the first coordinates by a coupling

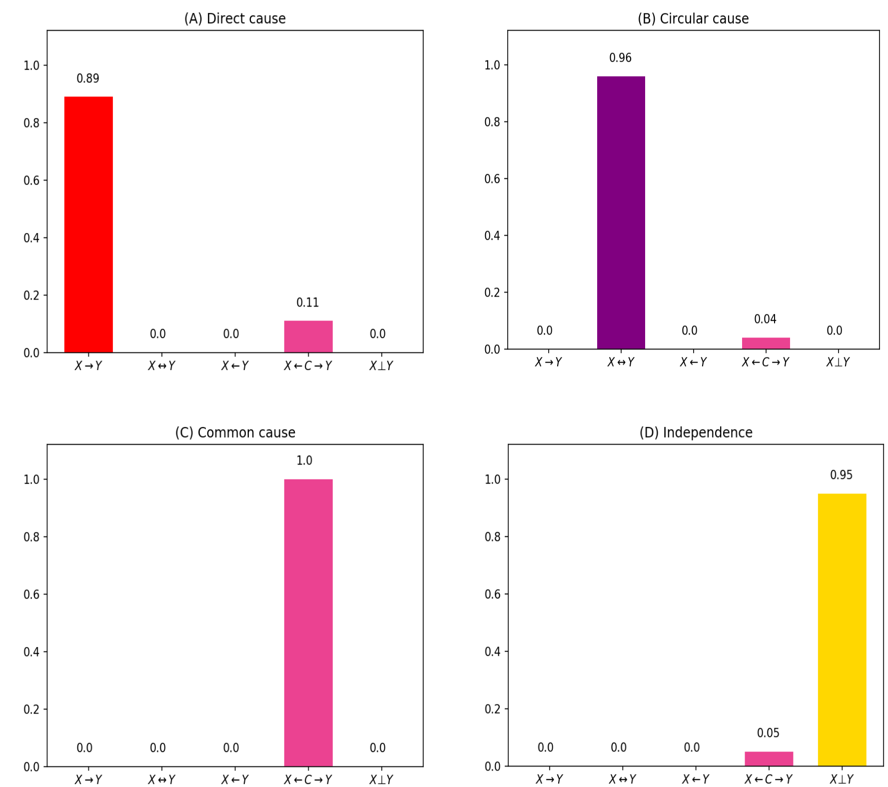
The system is defined by the following differential equations:

$$\dot{x}_1=\sigma(x_2-x_1)+m_{y\to x}(x_2-y_1)+m_{z\to x}(x_2-z_1)$$
$$\dot{x}_2=x_1(\rho-x_3)-x_2$$
$$\dot{x}_3=x_1x_2-\beta x_3$$

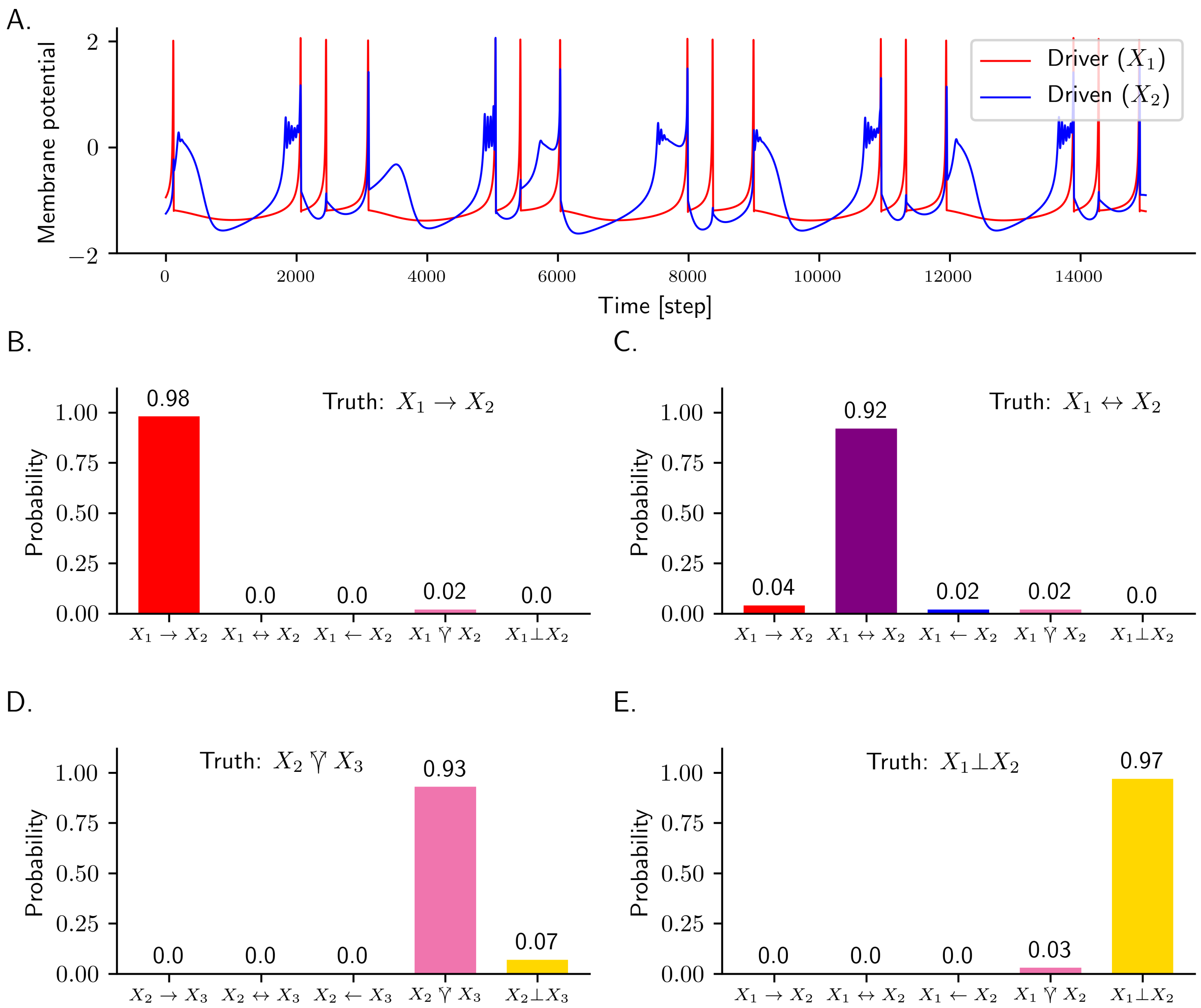
$$\dot{y}_1=\sigma(y_2-y_1)+m_{x\to y}(y_2-x_1)+m_{z\to y}(y_2-z_1)$$
$$\dot{y}_2=y_1(\rho-y_3)-y_2$$
$$\dot{y}_3=y_1y_2-\beta y_3$$

$$\dot{c}_1=\sigma(c_2-c_1)$$
$$\dot{c}_2=c_1(\rho-c_3)-c_2$$
$$\dot{c}_3=c_1c_2-\beta c_3$$

Causal relation probabilities

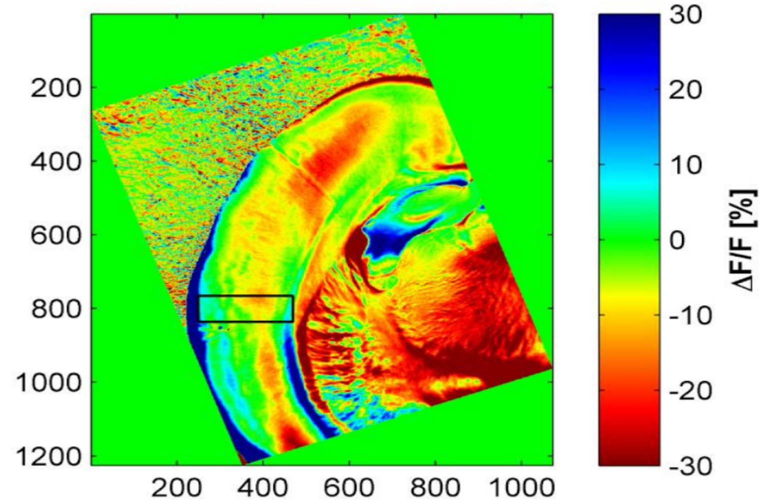


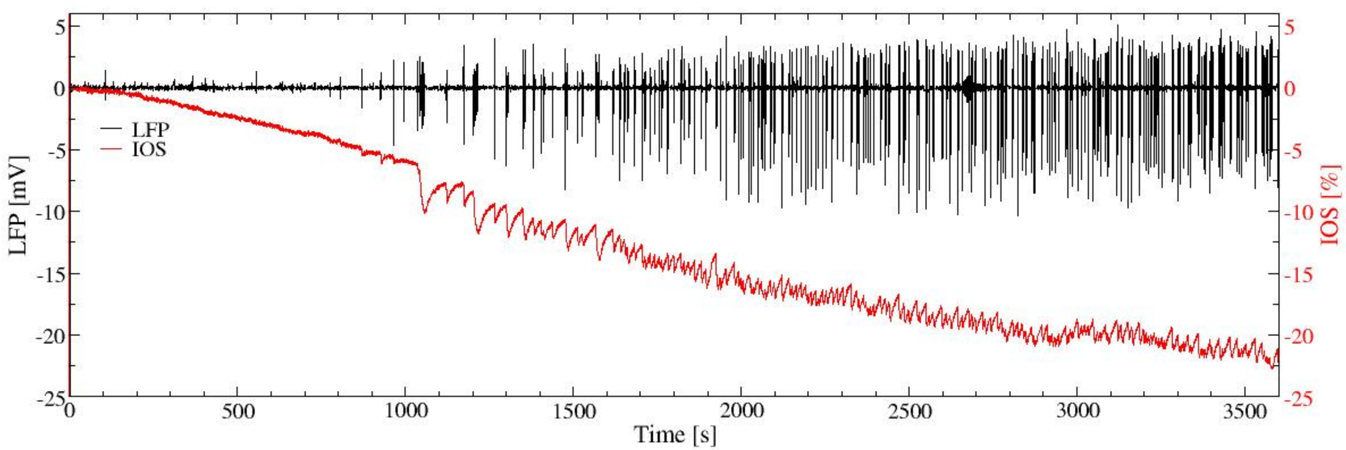
Test III: Hindmarsh-Rose model



Local Field Potential (LFP) vs Intrinsic Optical Signal (IOS)

Epileptiform activity was evoked by low Mg+ environment in vivo slice preparation. The local field potential was recorded together with the intrinsic optical signal (IOS), which is possibly a result of swelling of cells during over excitation.





During the long (1 hour) recording, epileptiform bursts appeared with increasing frequency. Parallel, the optical reflectance (and the transmittance) of the tissue changes for visible light, without any additional dying. The process is clearly activity dependent, but slow.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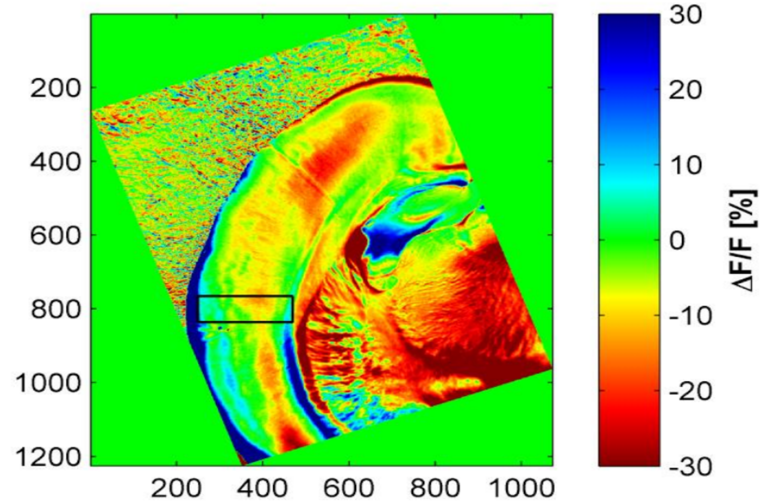
LFP vs IOS

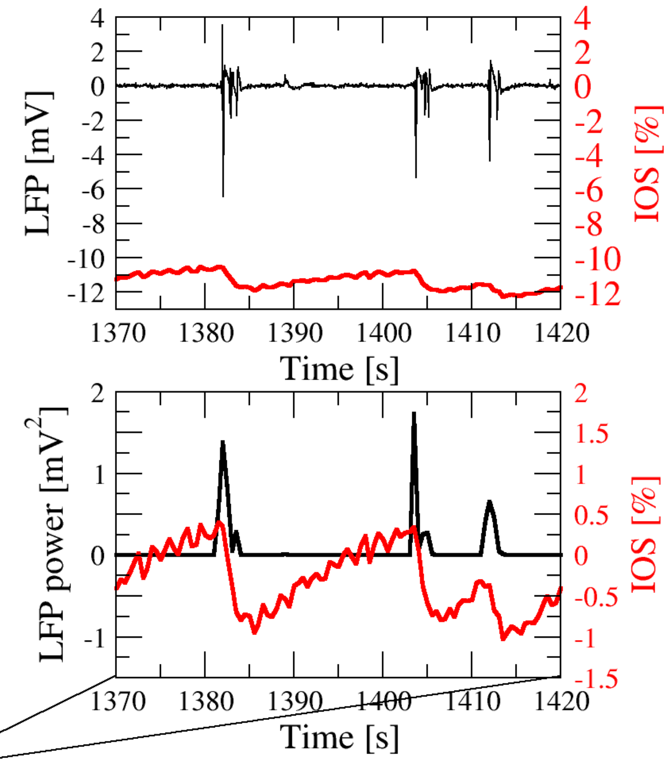
The sampling frequency of the IOS was only 2Hz, much lower than the 1kHz of the LFP!!!

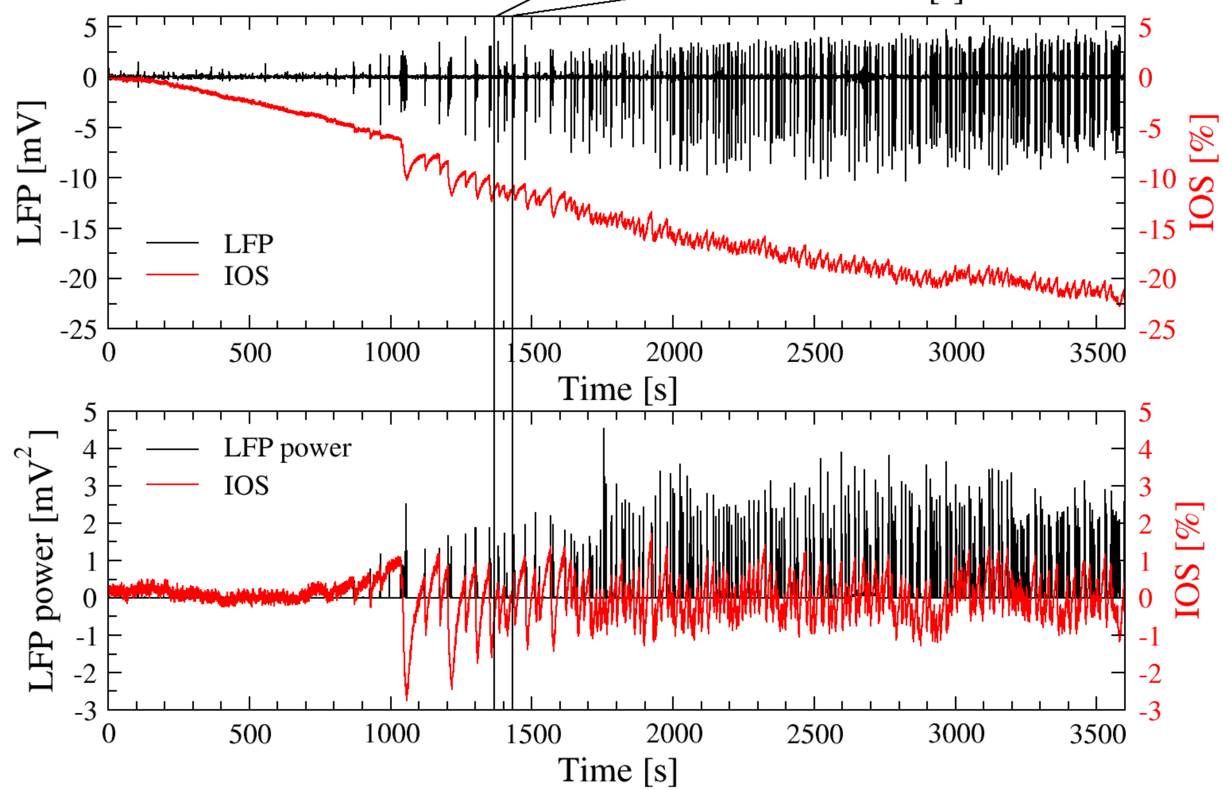
In order to make the causality analysis applicable:

The faster and slow component of the IOS were divided by subtracting a moving window average,to get stationary time series.

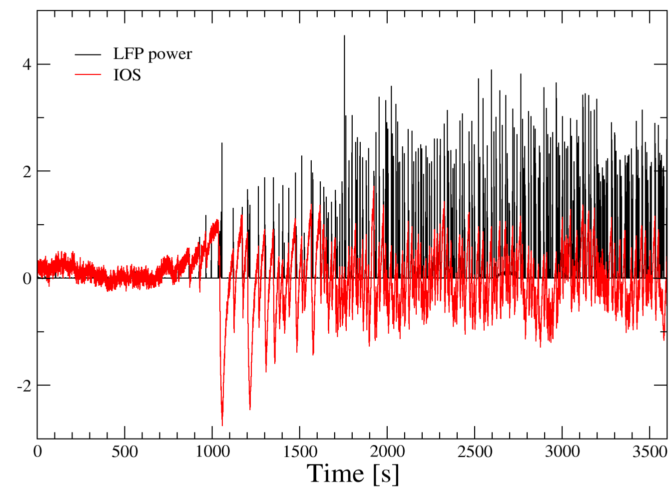
The LFP has been downsampled by summing up the V^2 for every 500 ms

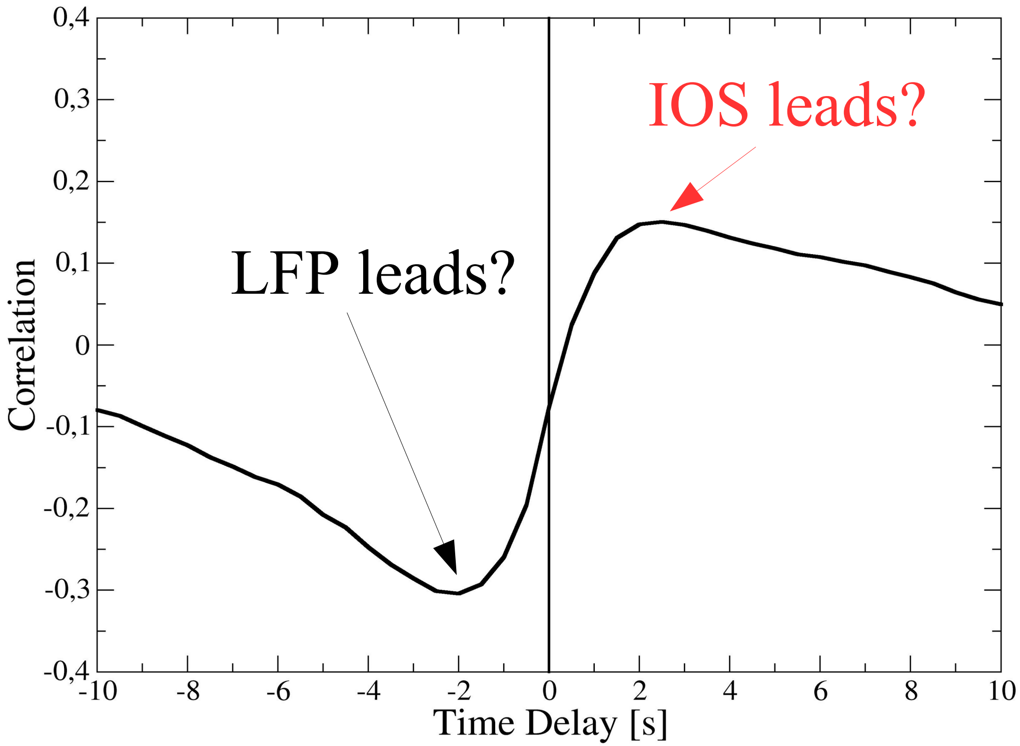






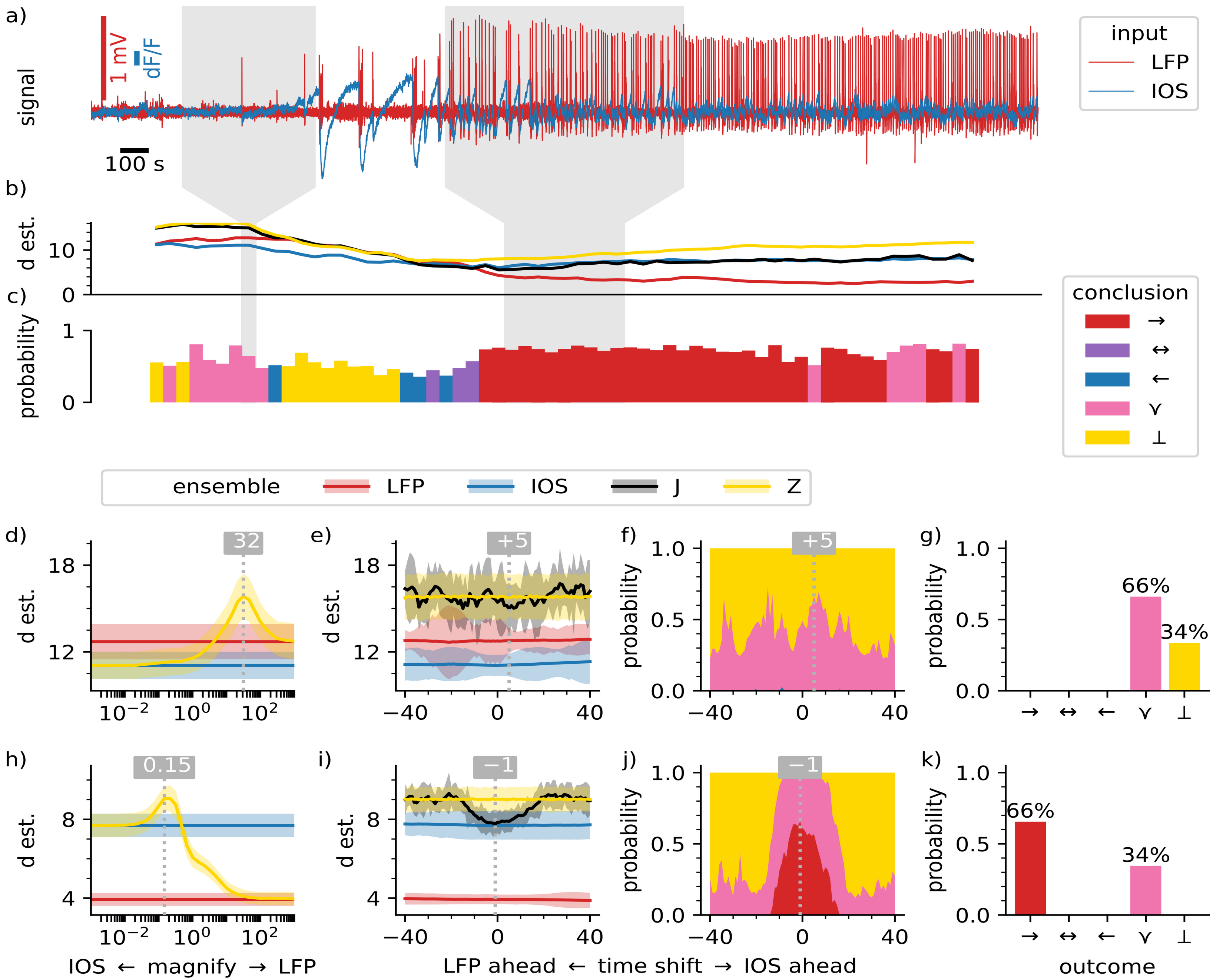
LFP-IOS cross correlation





The instantaneous correlation is nearly zero, the cross correlation function has two significant peaks: a higher negative one at -2s (LFP leads) and a smaller positive one at +2.5s (IOS leads). This could be the sign of a well delayed interaction.

Application of Dimensional Causality to evoked seizure-like activity in vitro

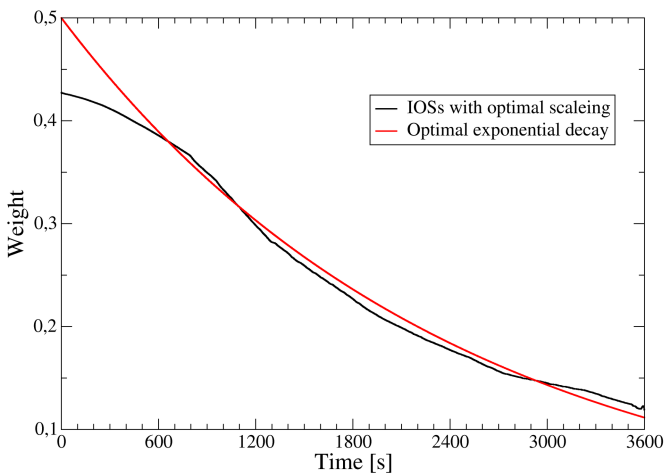


Reverse engineering

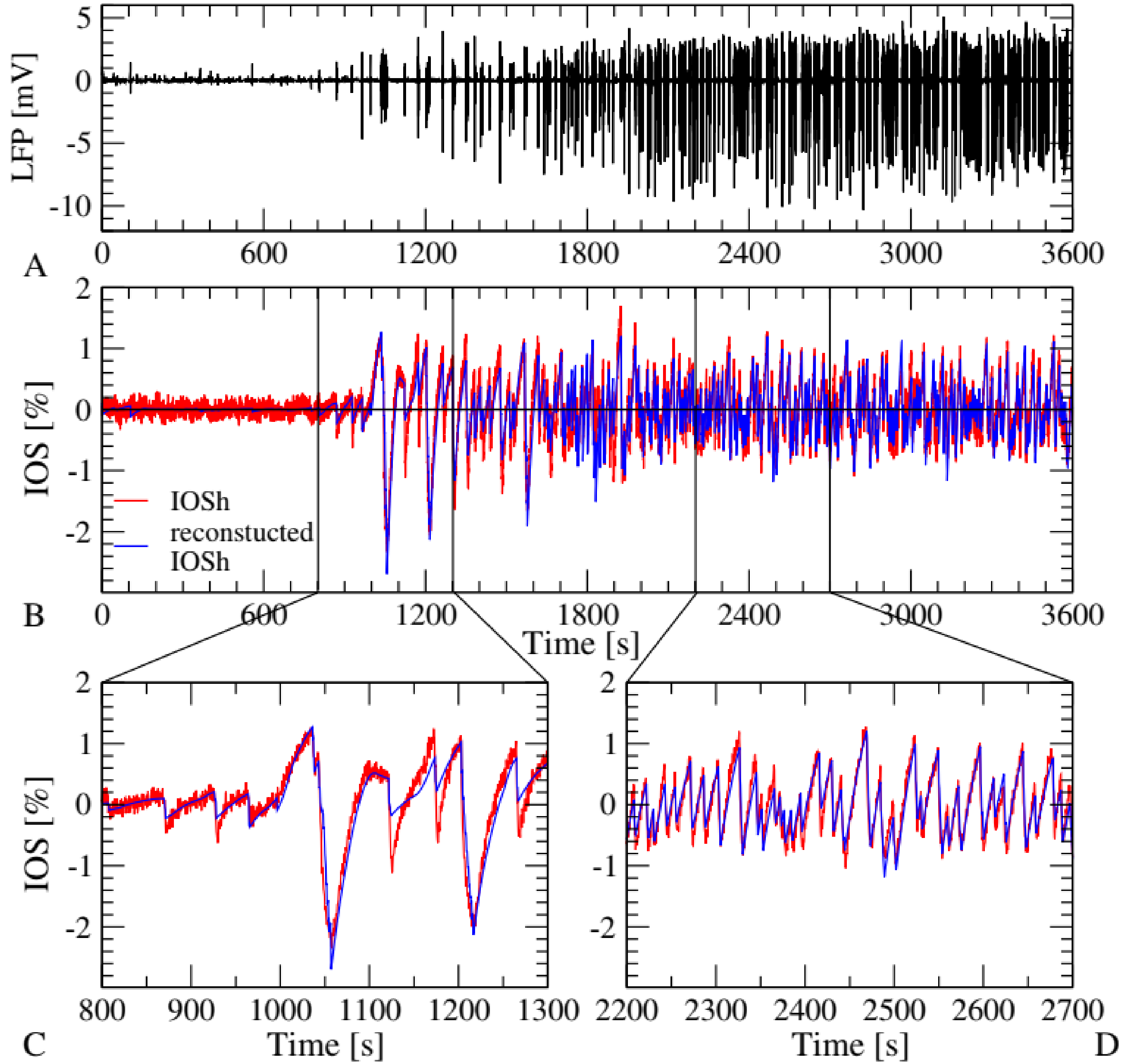
$$\frac{dIOSh}{dt} = W(t) * LFP^2 - \frac{IOSh(t)}{\tau_1}$$

Where:

$$W(t) = W_0 * e^{\frac{-t}{\tau_2}}$$



The IOS time series was reconstructed, based on the LFP recording with high precision during the 1h long session.



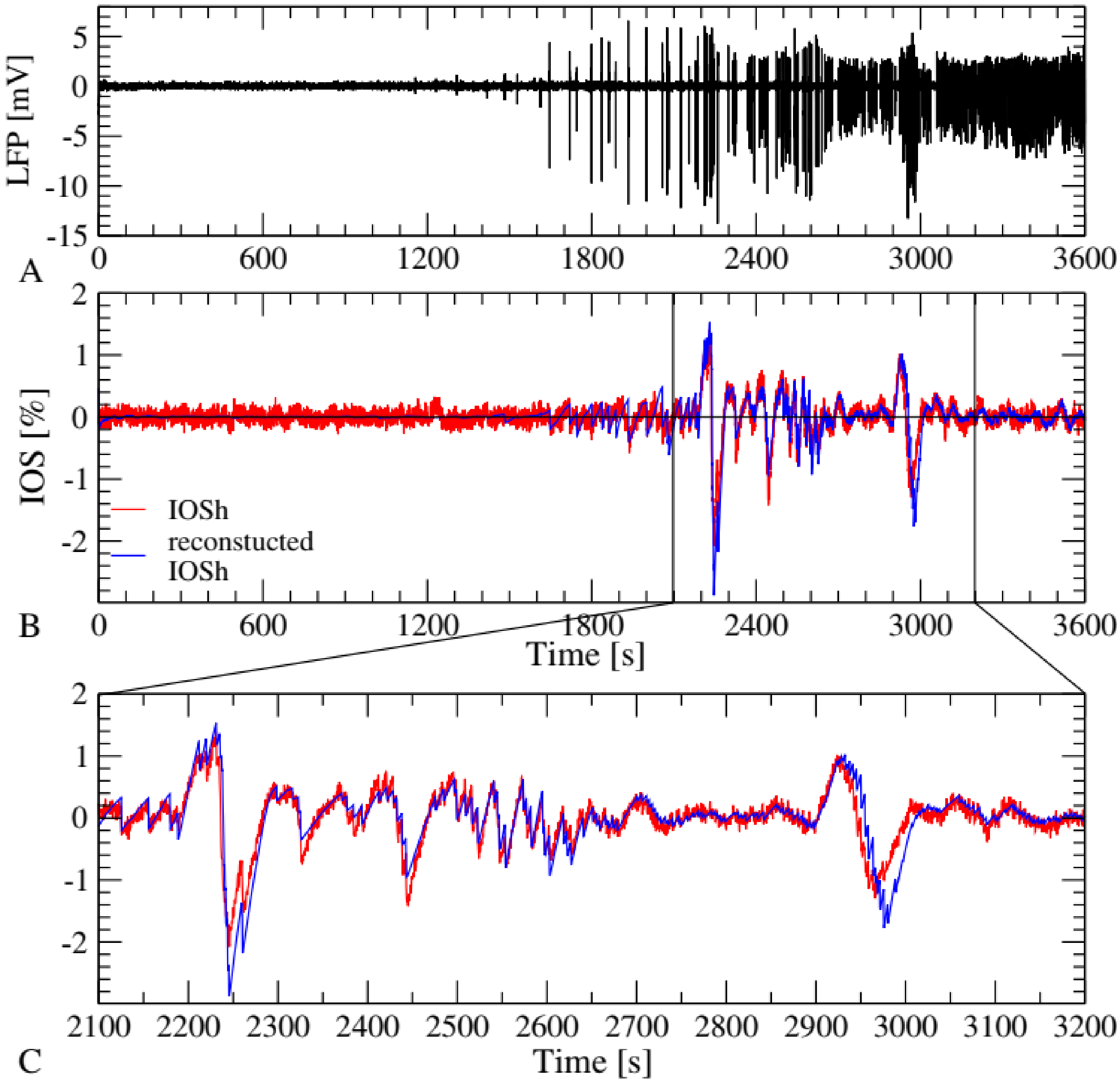
Reverse engineering

$$\frac{dIOSh}{dt} = W(t) * LFP^2 - \frac{IOSh(t)}{\tau_1}$$

Where:

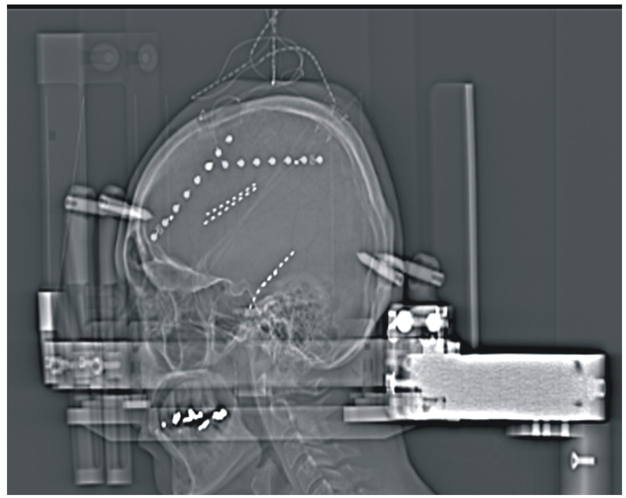
$$W(t) = W_0 * e^{\frac{-t}{\tau_2}}$$

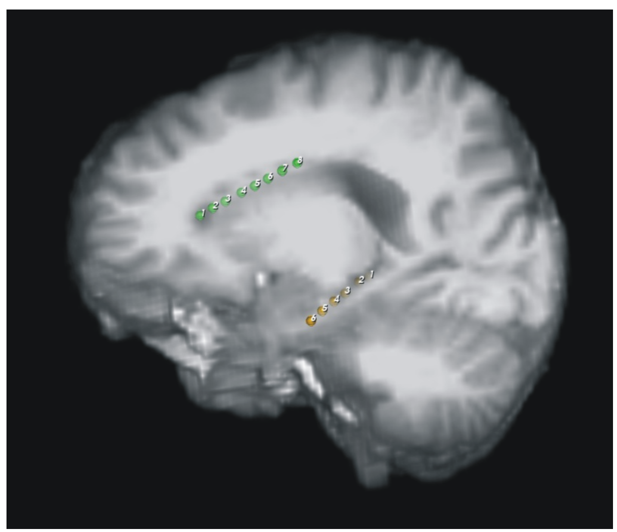
The same model, with different parameters describes the 4AP activity as well.



Intra- and inter hippocampal connectivity during seizure

In order to find out the lateralization of the seizure onset, two near-hippocampal electrodes inserted through the foramen ovale into the lateral ventricles.





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Virág Bokodi

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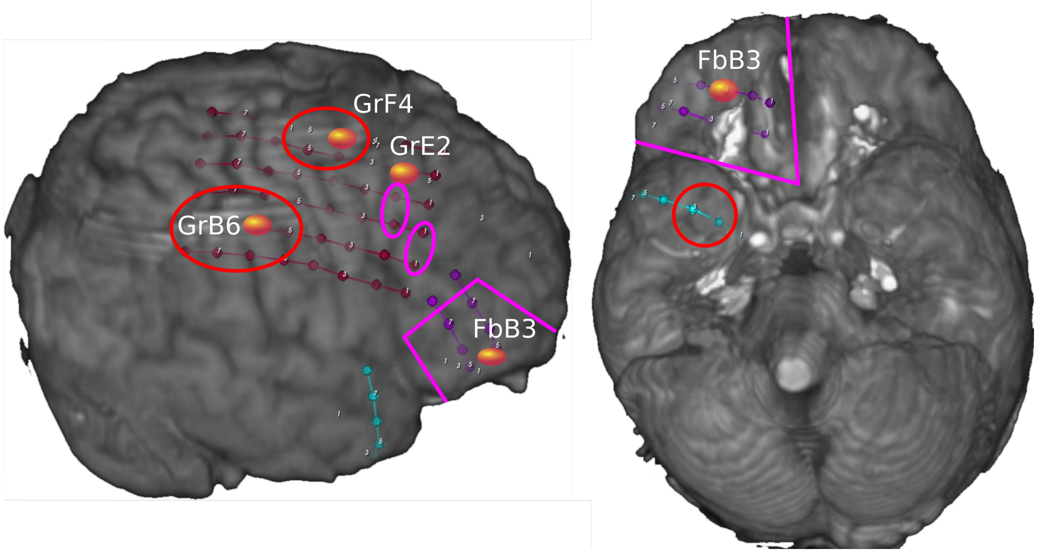
National Institute of Clinical Neurosiences

Application: localization the origin of the epilepsy

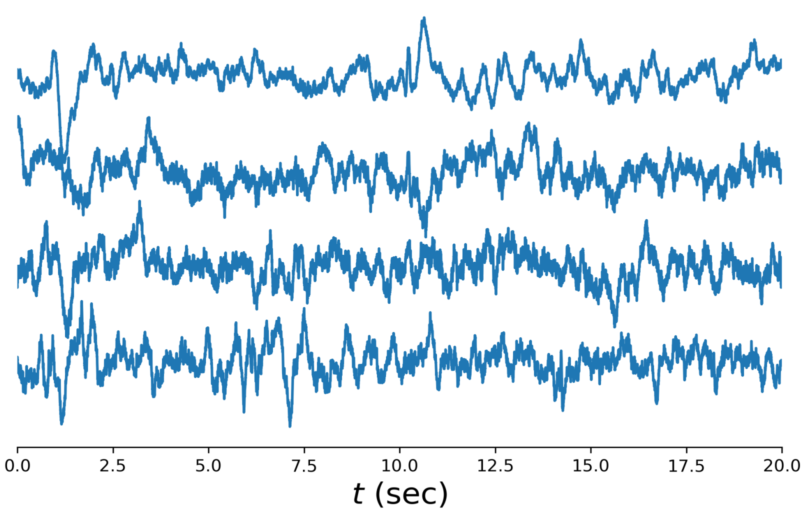


The 20-year-old patient suffered from a drug resistant epilepsy with frequent seizures.

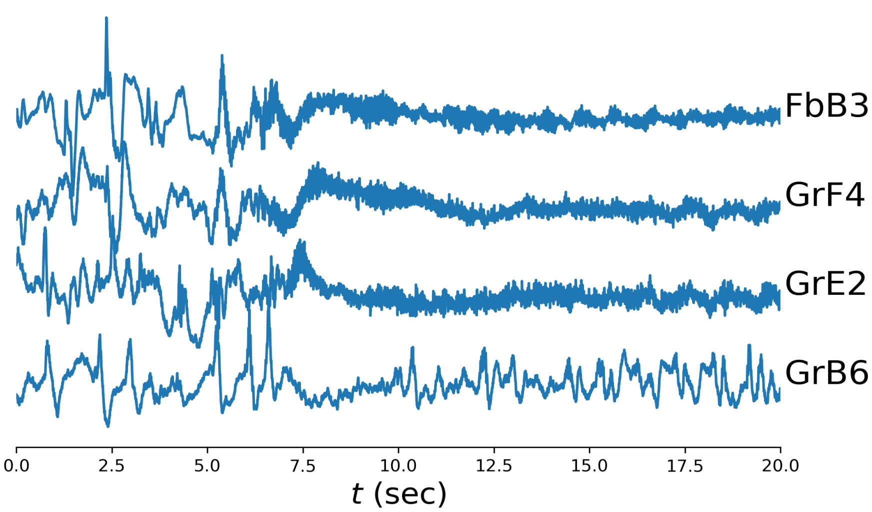
The finding of a cortical dysplasia (at GrF4 electrode site) raised the possibility of the surgical treatment

GrB6 and GrF4 were only slightly involved (red ellipses). Based on the pronounced seizure activity, and the sensitive position of GrB6, only the frontal and orbitobasal parts were cut (purple signs).

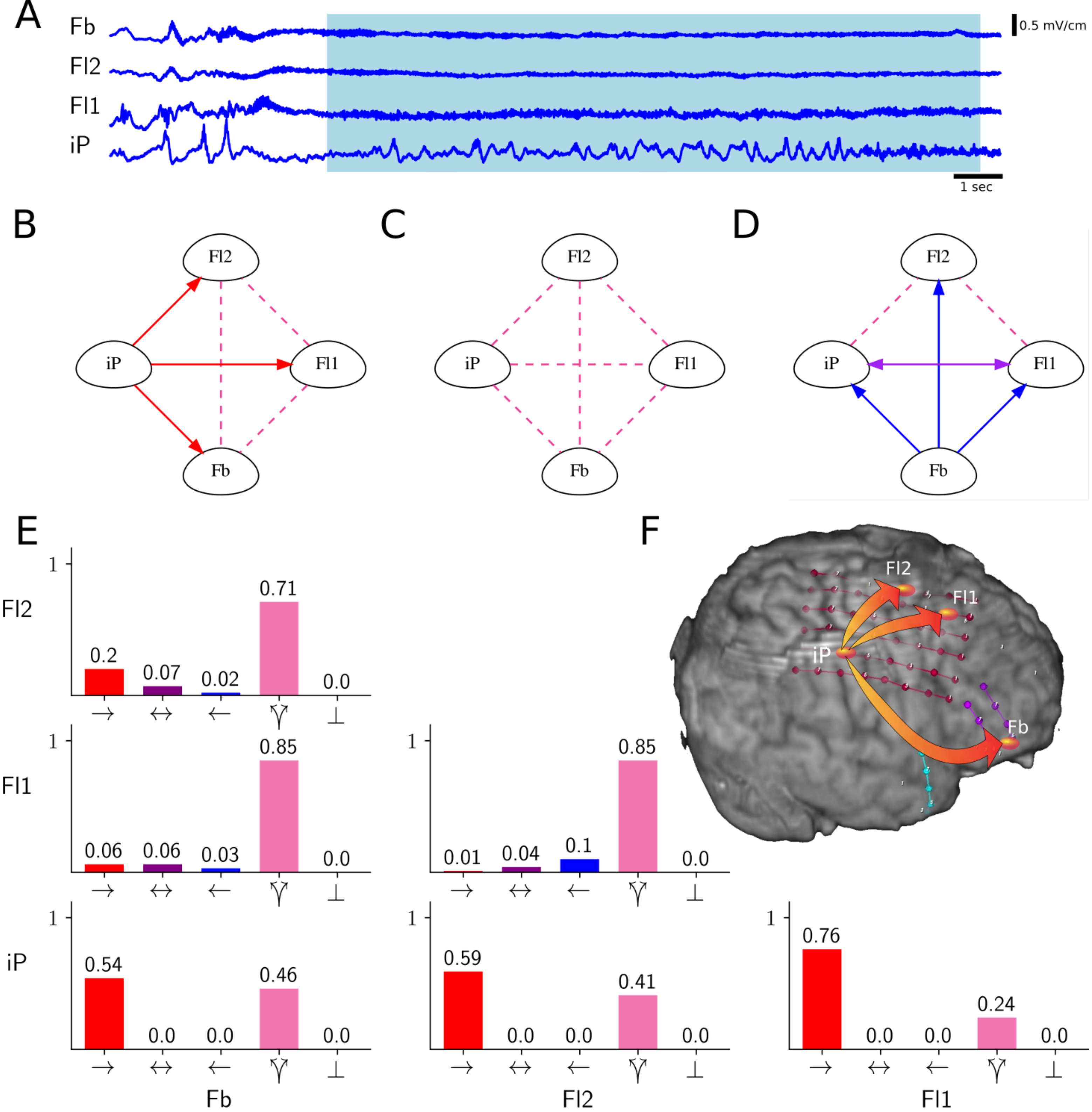
Interictal



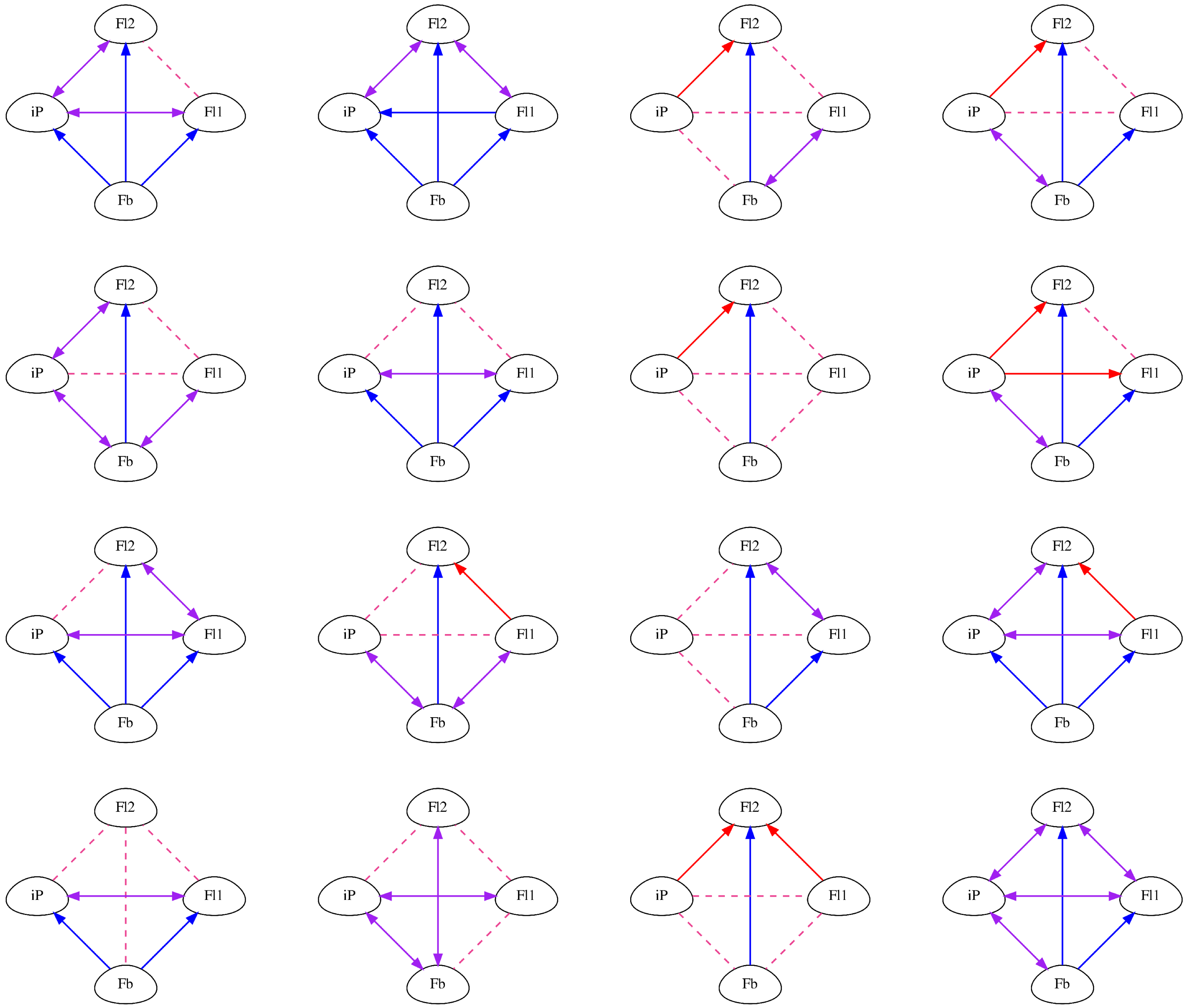
Seizure



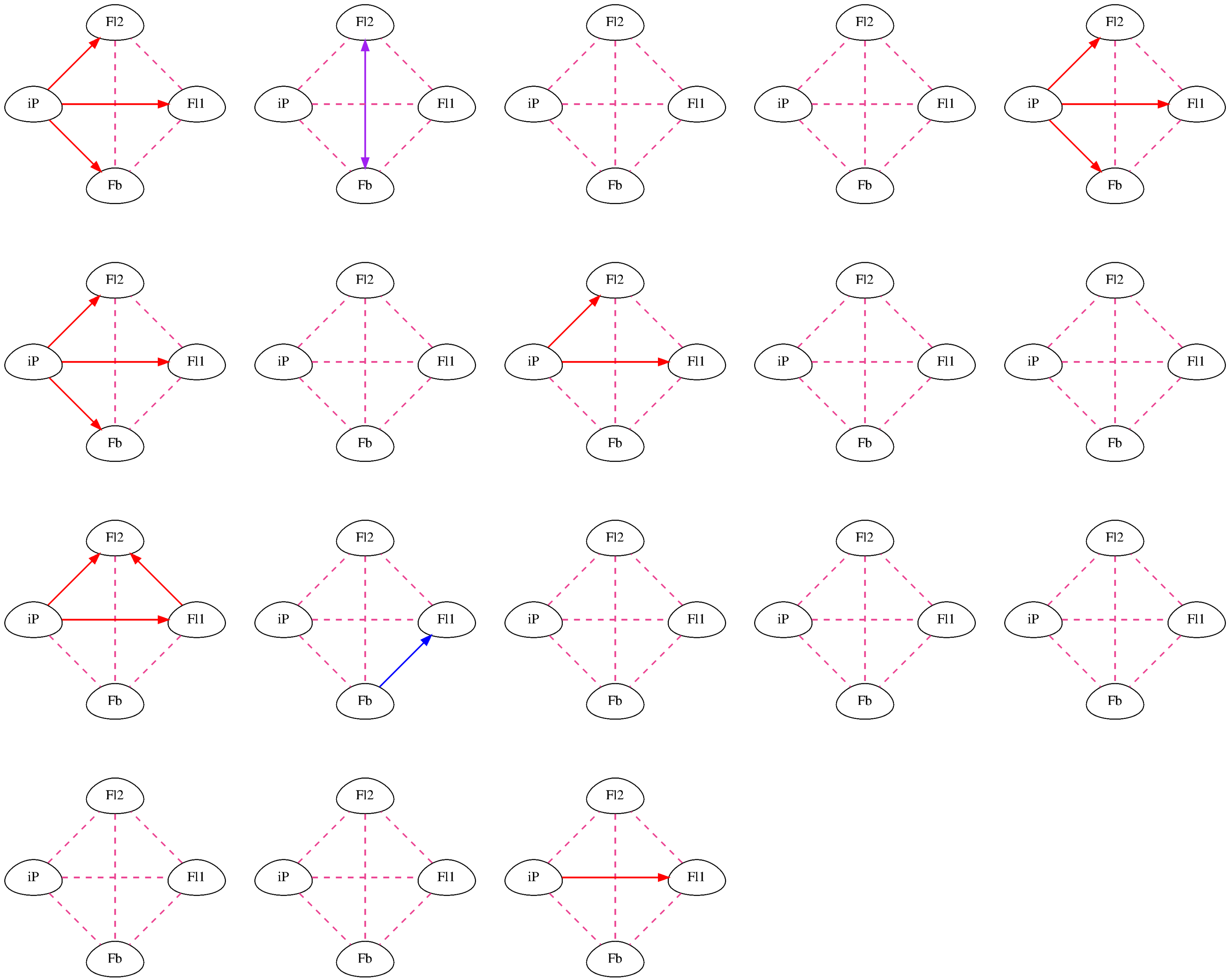
Application: localization the origin of the epilepsy



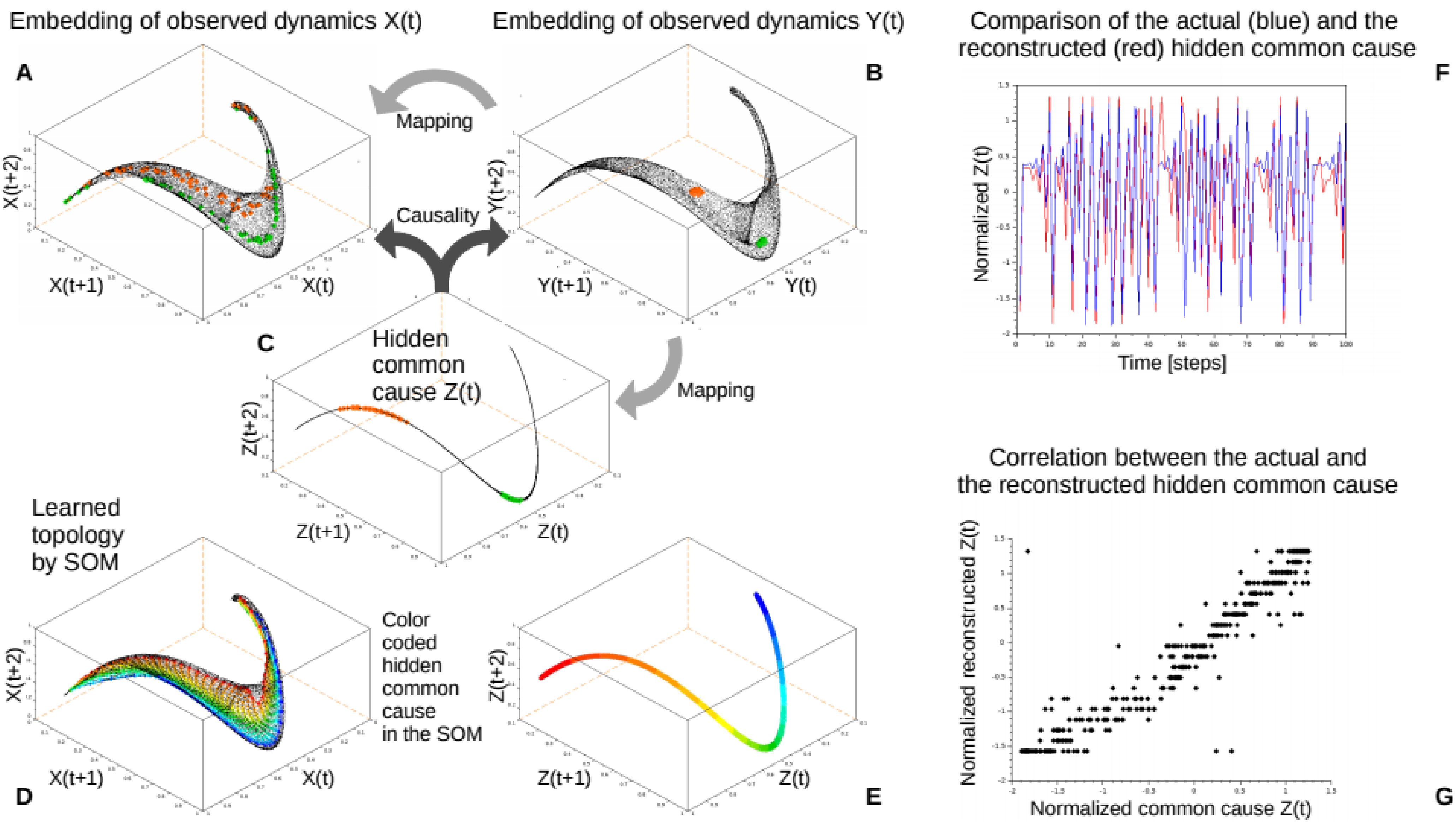
Interictal periods



Multiple seizures

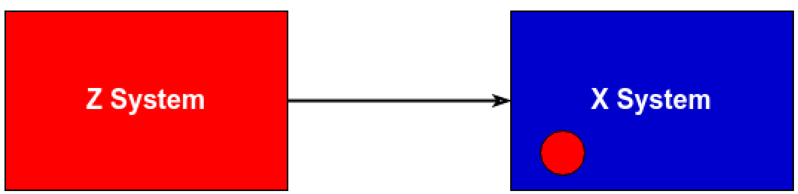


Reconstruction of the hidden common cause



Coupled dynamical systems and shared dynamics

Unidirectional coupling

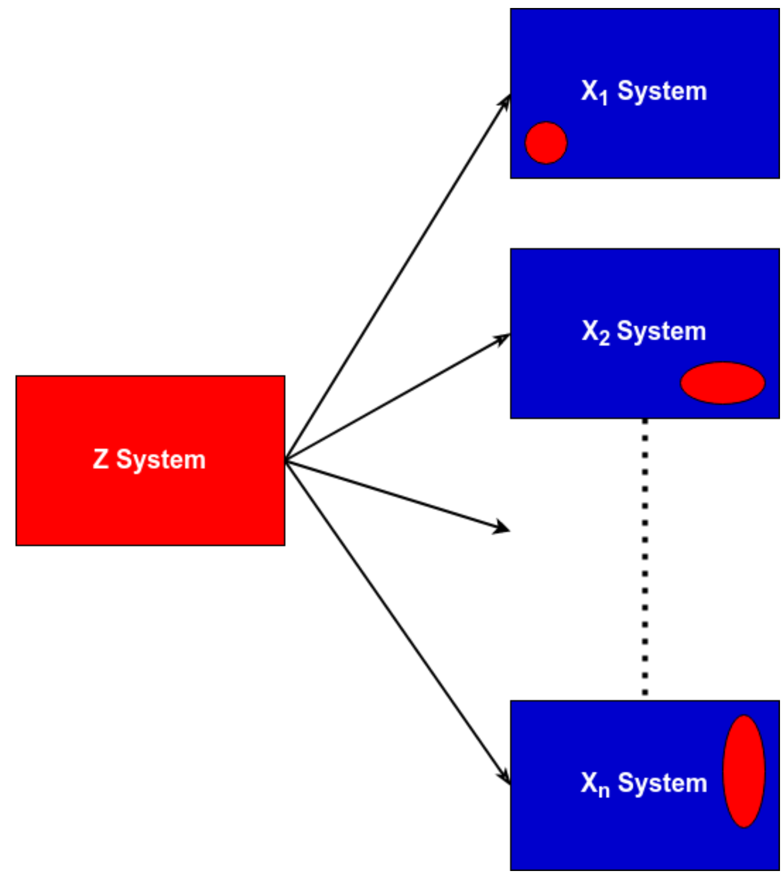


Causal effect leaves a mark on the dynamics

A common cause means shared dynamics in the effects

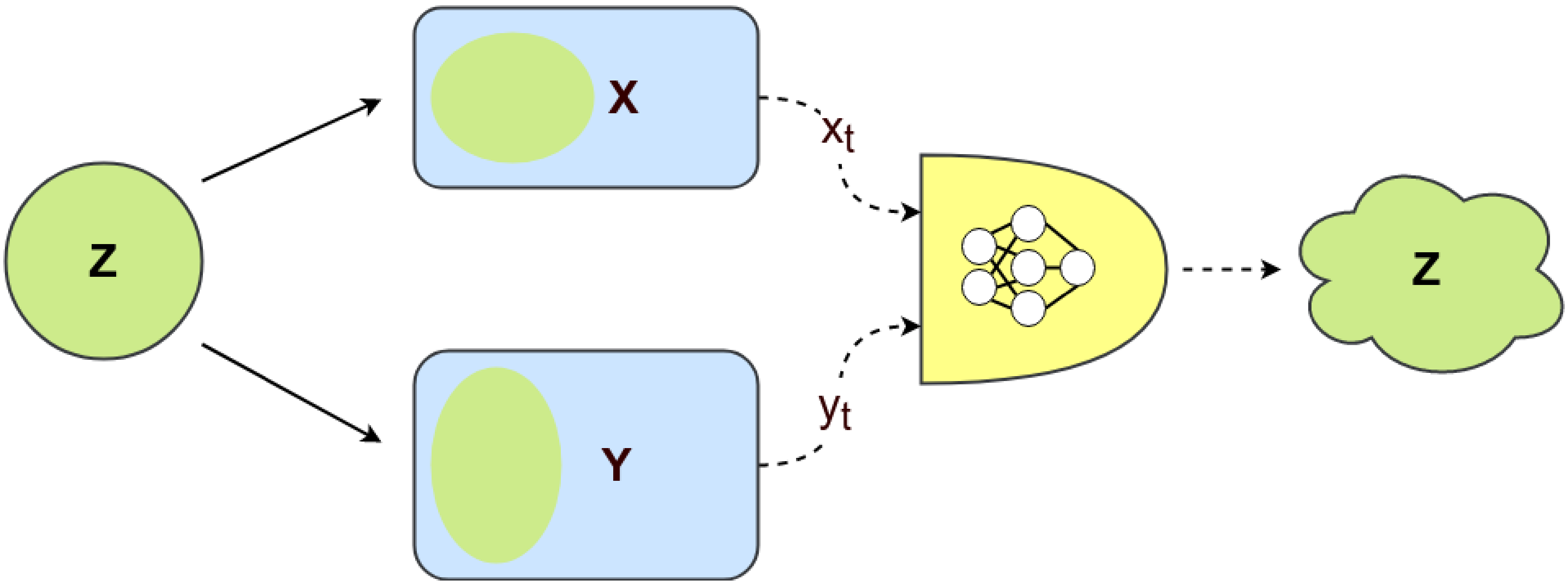
Possibility of decomposition!

Common cause



Stark('99), Sauer(2004), Wiskott(2003), Sugihara(2012), Benko et al(2018)

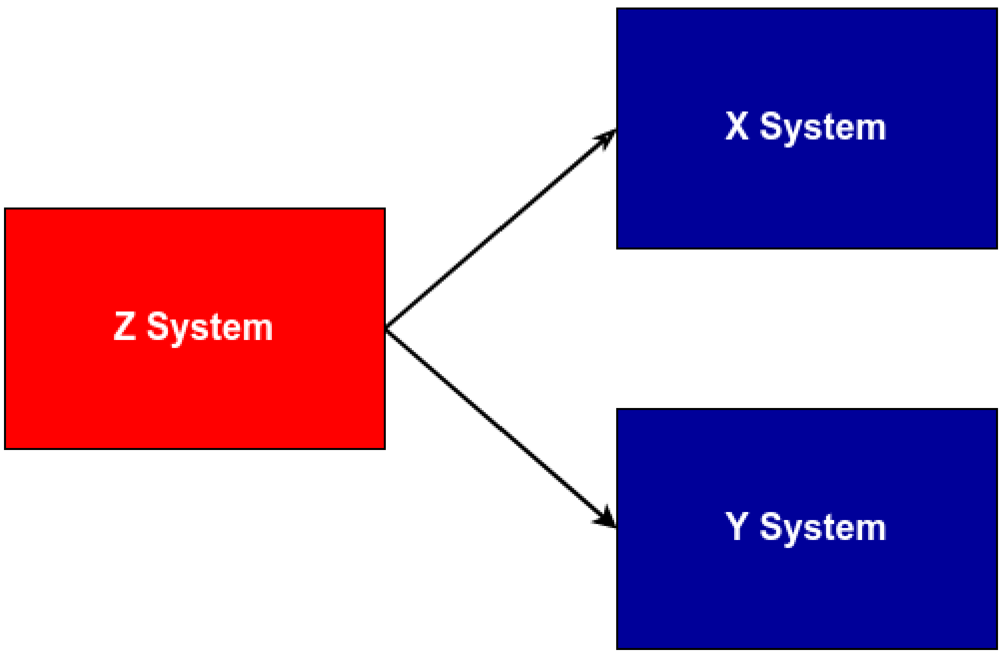
Coupled dynamical systems and shared dynamics



Goal: Reconstruct Z!

Example system: Coupled Logistic maps

Observed



$$x_t = r x_{t-1}(1 - x_{t-1} - \beta_{xz} z_{t-1})$$

$$y_t = r y_{t-1}(1 - y_{t-1} - \beta_{yz} z_{t-1})$$

$$z_t = r z_{t-1}(1 - z_{t-1})$$

Unobserved

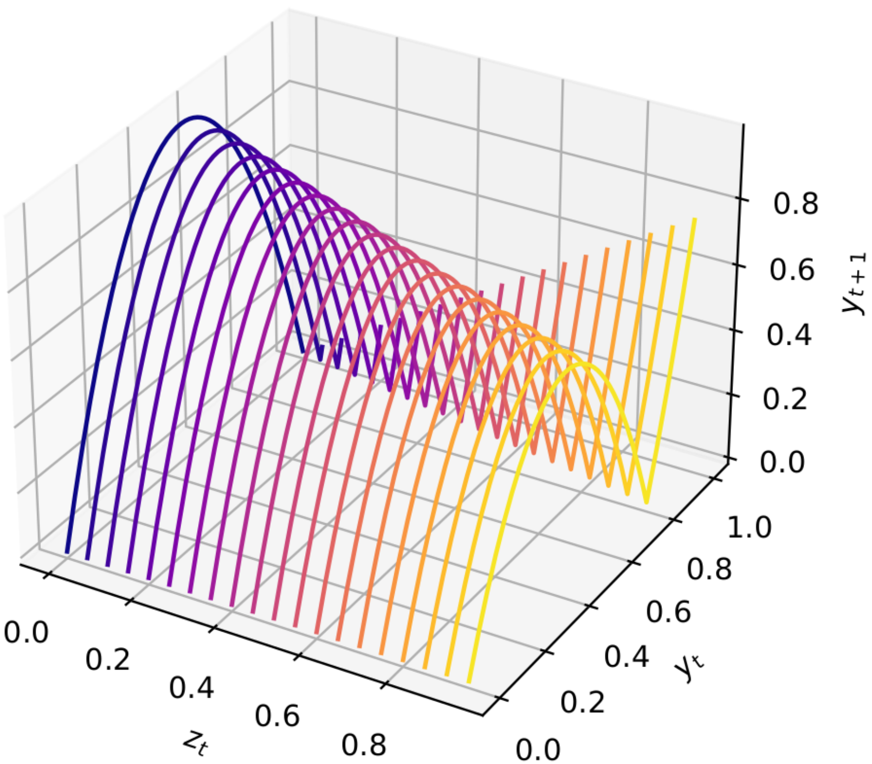
Goal: Reconstruct Z!

Mapping from Embedding-space to state-space

$$f: \quad x_t = r x_{t-1}(1 - x_{t-1} - \beta_{xz} z_{t-1})$$

$$\boxed{g: \quad y_t = r y_{t-1}(1 - y_{t-1} - \beta_{yz} z_{t-1})}$$

$$h: \quad z_t = r z_{t-1}(1 - z_{t-1})$$

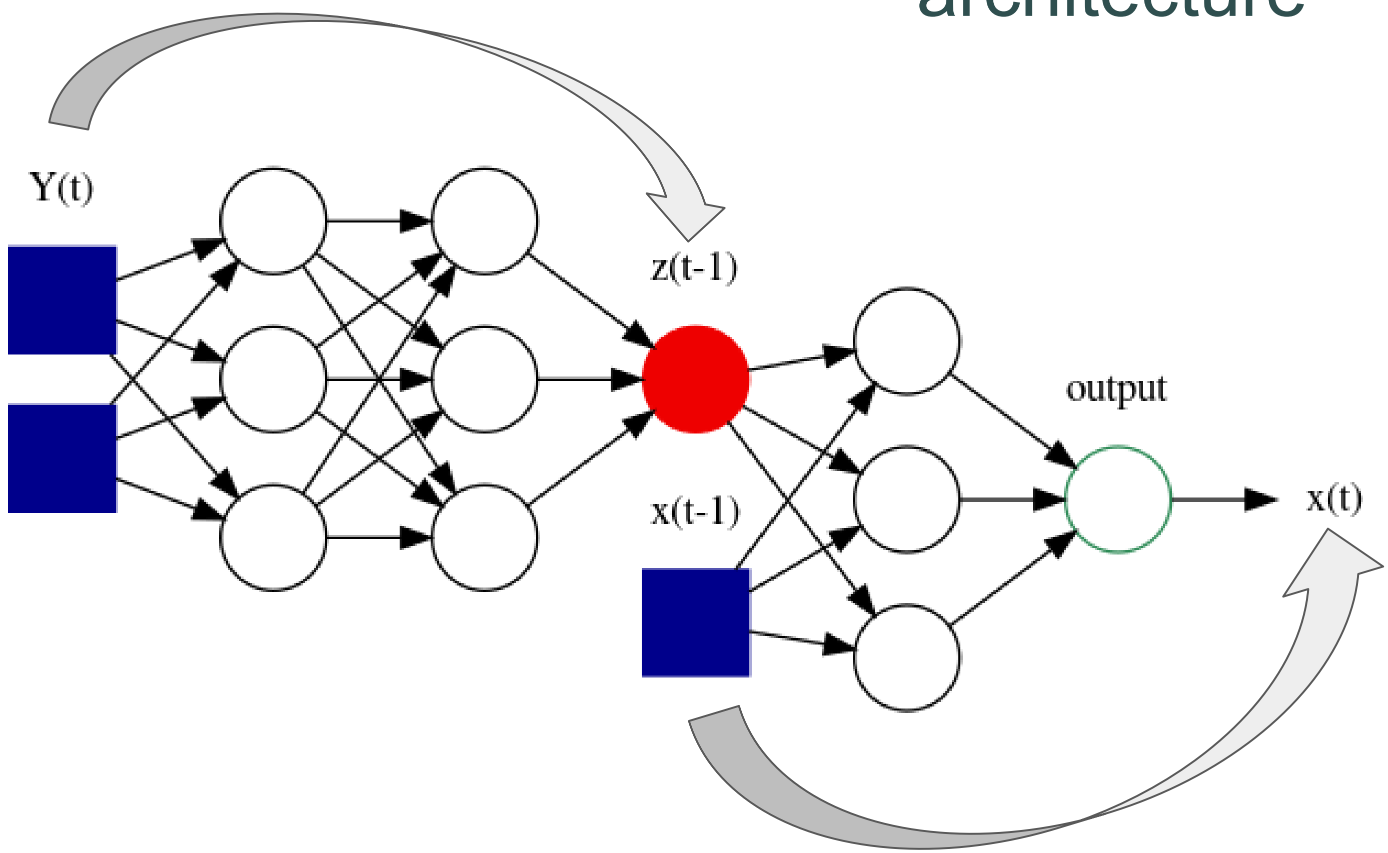


$$Y_t = [y_t, y_{t-1}] \longrightarrow S_t = [y_{t-1}, z_{t-1}]$$

$$\phi: \quad z_{t-1} = \frac{1 - y_{t-1} - \frac{y_t}{r y_{t-1}}}{\beta_{yz}}$$

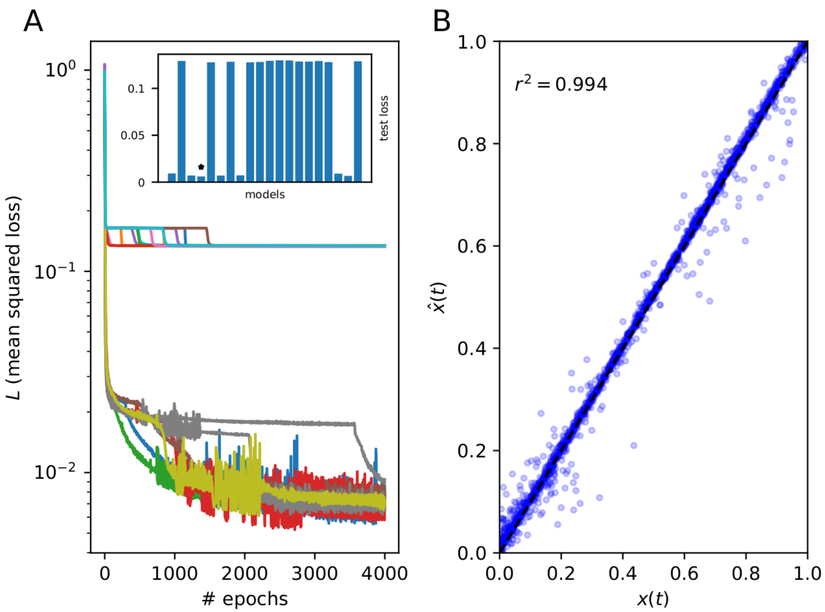
Approximate the mapping with a Neural Network

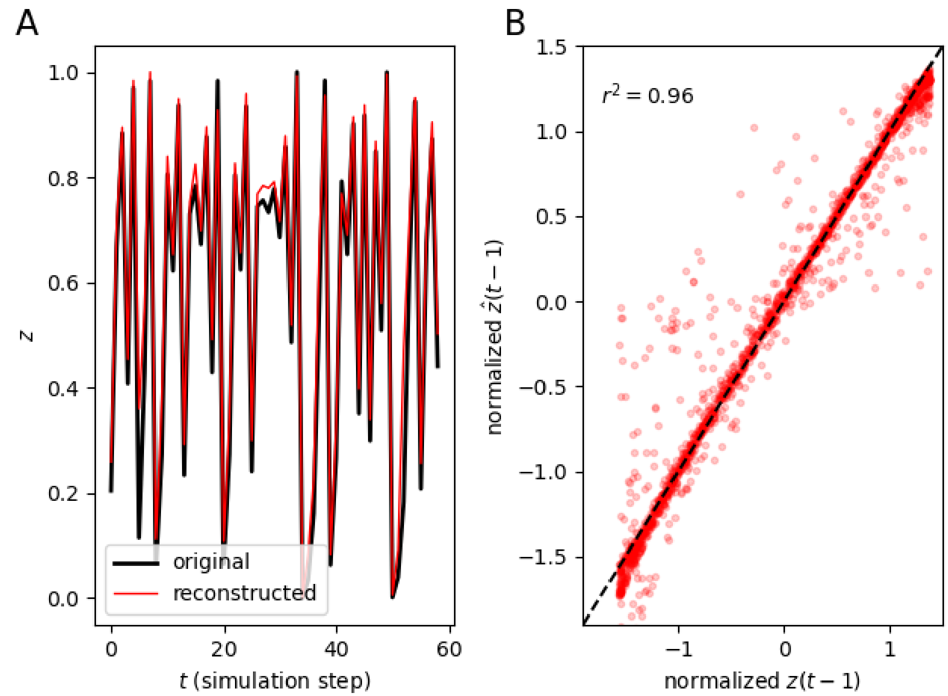
The mapper-coach architecture



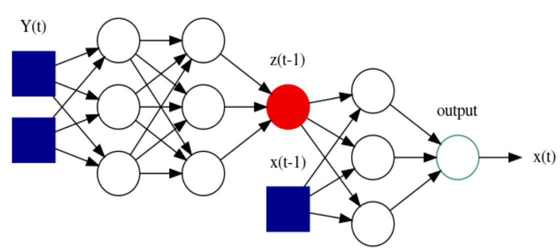
Revealing the latent dynamics by ANNs

Hidden unit vs. latent dynamics:





$$\mathcal{L} = RMSE(X)$$



Benkő, Z., & Somogyvári, Z. (2021). *Reconstructing common latent input from time series with the mapper-coach network and error backpropagation*. *3*, 1–7. http://arxiv.org/abs/2105.02322

Theoretical Neuroscience and Complex Systems Research Group

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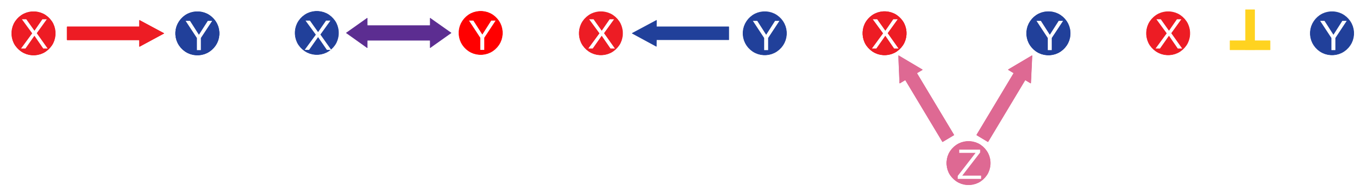
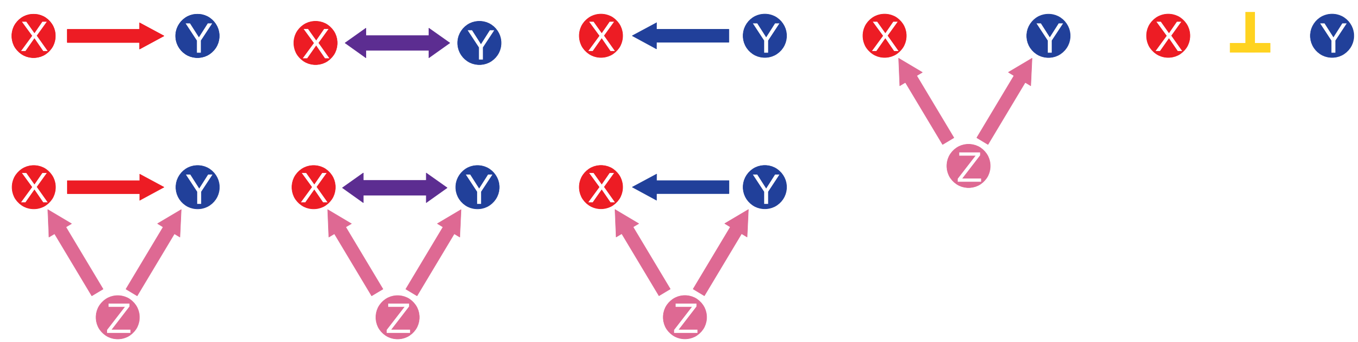
Thank you for your attention!

Dimensional Causality equivalence classes

Dimensional Causality



Truth
by considering only weak direct connections

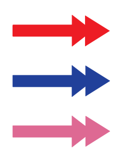


Dimensional Causality equivalence classes

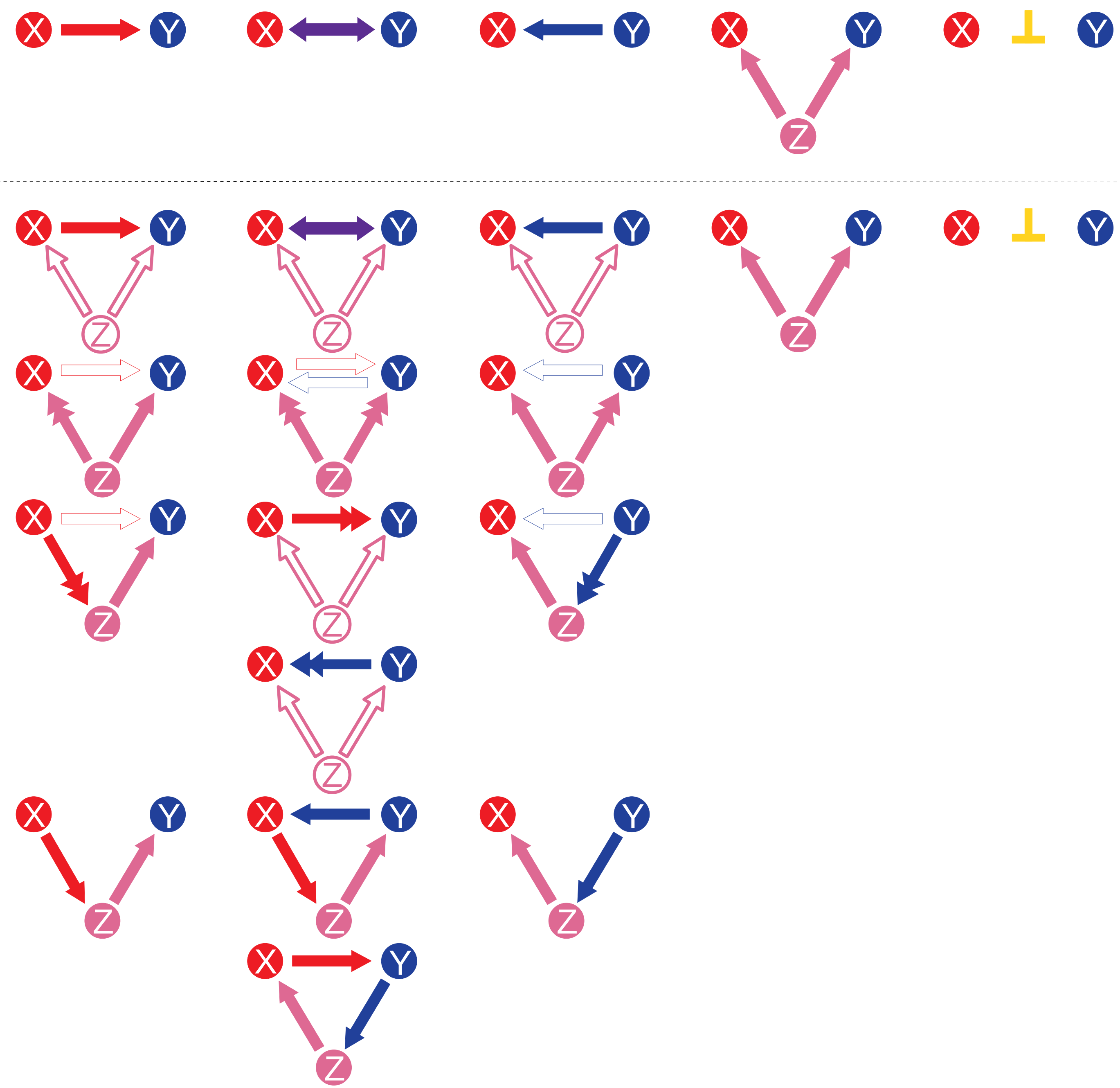
Dimensional Causality

Truth

Allowing strong connections causing generalized synchrony:



Considering indirect connections:



Recent publications on these topics

- Zsigmond Benkő, Tamás Bábel, Zoltán Somogyvári: Model-free detection of unique events in time series. Scientific Reports 12:227 (2022) doi:10.1038/s41598-021-03526-y.
- Zsigmond Benkő, Marcell Stippinger, Roberta Rehus, Attila Bencze, Dániel Fabó, Boglárka Hajnal, Loránd Erőss, András Telcs, Zoltán Somogyvári: Manifold-adaptive dimension estimation revisited, PeerJ Computer Science (2022): 8:e790 https://doi.org/10.7717/peerj-cs.790.
- Ádám Zlatniczki, Marcell Stippinger, Zsigmond Benkő, Zoltán Somogyvári, András Telcs: Relaxation of Some Confusions about Confounders Entropy 23 (11), 1450, 2021
- Zsigmond Benkő, Zoltán Somogyvári: Reconstructing common latent input from time series with the mapper-coach network and error backpropagation, arXiv:2105.02322 (2021).
- Zsigmond Benkő, Kinga Moldován, Katalin Szádeczky-Kardoss, László Zalányi, Sándor Borbély, Ildikó Világi, Zoltán Somogyvári: Causal relationship between local field potential and intrinsic optical signal in epileptiform activity in vitro Scientific Reports 9 Article number: 5171 (2019)
- Zsigmond Benkő, Ádám Zlatniczki, Marcell Stippinger, Dániel Fabó, András Sólyom, Loránd Erőss, András Telcs, Zoltán Somogyvári: Complete Inference of Causal Relations between Dynamical Systems, arxiv:/1808.10806 .