Foundations of Mathematics, Lecture 8

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BMETE91AM35 Fall 2022-23

Plan of this class

- First half of class: more on languages
- Second half of class: Midterm. Please put your NAME and NEPTUN code on each page you submit. Within the next 24 hours (Oct 26) you can submit LaTeXversion of your solutions for extra points (but you can get 100% without this step)

Rudiments of formal language theory

- Given an alphabet Σ, the set of all strings formed from these is denoted Σ^*. There is a special element λ called the *empty string*.
- Length of λ is 0, length of $a \in \Sigma$ is 1, length of α denoted $|\alpha|$ satisfies $|\alpha\beta| = |\alpha| + |\beta|$
- The main operation on strings is *concatenation* (writing them is sequence). For example, if $\alpha = abc$ and $\beta = AB$ then $\alpha\beta = abcAB$
- Concatenation is *not* commutative, $\beta\alpha = ABabc \neq \alpha\beta$
- We abbreviate $\alpha\alpha$ as α^2, similarly for α^3 etc.
- A **language** over the alphabet Σ is a subset of Σ^*
- Since languages are sets, it is meaningful to speak of their union, intersection, and complement (relative to Σ^*)
- The **product** of languages R and S, written RS, is $\{\alpha\beta | \alpha \in R, \beta \in S\}$
- The set $\cup_{i=0}^{\infty} R^i$ is written R^* and is called the **Kleene closure** of R.

Rudiments of automata theory

- Finite state automata
- Turing machines