

FOUNDATIONS OF MATHEMATICS, LECTURE 1

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ABOUT THIS COURSE

- This is a first (foundational) course, there are no prerequisites
- The goal is to learn how mathematics is done: what are axioms, what are statements (lemmas, theorems) and how we prove these
- Students will learn about ‘foundational’ subjects: set theory, logic, category theory, algorithms – mostly by solving problems
- Students will also learn some practical skills such as \LaTeX , bibtex, etc.
- Class webpage at <https://kornai.com/2022/FoundationsOfMathematics>
- Textbook ChartrandPolimeniZhang.pdf available from this page

GRADING

- Prelim test (today) gives 0% of the grade!
- Weekly problem sets give 50% of the grade
- Class activity counts for 25%
- Final exam gives remainder of the grade (25%)

MATERIAL COVERED (NOT IN THIS ORDER)

- Intro set theory: sets, functions, relations, operations
- Intro logic: formulas, axioms, deduction, models
- Intro category theory: objects, arrows, functors, natural transformations
- Intro algorithms: input, output, computation
- Numbers: \mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{R} , \mathbb{C}
- Structures: groups, rings, fields, modules
- Probability: elementary foundations

WHY IS IT LIKE THIS?

- ① Historically, math grew out of two main threads: counting and geometry
- ② The foundations were initially Euclid's reaction to the Sophists (the same axiomatic method was used by Pāṇini to create the foundations of grammar)
- ③ Modern foundations were developed \sim 1850-1950

1. Three subsets A, B, and C of $\{1,2,3,4,5\}$ have the same cardinality. Furthermore,

A 1 belongs to A and B but not to C

B 2 belongs to A and C but not to B

C 3 belongs to A and exactly one of B and C

D 4 belongs to an even number of A, B, and C

E 5 belongs to an odd number of A, B, and C

F The sums of the elements in two of the sets A, B, and C differ by 1

What is B?

2. (a) Compute $\cos(20^\circ) \cos(40^\circ) \cos(80^\circ)$

(b) Prove that the result is exact

3. $\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}$ – find all integer solutions