A matematika alapjai, 7. előadás

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BMETE91AM35 2021-22 Őszi Félév

Organization of this class

- 2:15 – 13:00 ZH megbeszélése
- 13:00 – 13:45 Előadás folyt.
- 28 fős évfolyam, ebből 22 adott be, 16 LaTeX-et is, 5 angolul.
- 6 egyes (akik be se adtak), különben a legrosszabb jegy a hármas (5 fő), 7 db négyes, 10 ötös.
- Megoldókulcs `sol1.pdf` alatt
- Subject: MATALAP **NEPTUN** HF **n**
  pdf csatolva

Logic

- To define a logic we will need four things:
- A language to write formulas
- A notion of truth
- A notion of what the formulas mean ‘model theory’
- A deduction procedure ‘proof theory’
- We will discuss three main varieties: propositional, first order, and higher order logic
- We begin at the middle, even though both propositional and higher-order systems are substantially simpler

First order language

- It is convenient to use a very large (transfinite) list of *constants*. These are the things we want to talk about (points on the plane, sets, etc.)
- We also permit an infinite, but denumerable list of *variables* $x, y, z, \ldots$ to help us talk about many things at the same time
- Relation symbols, each with a fixed **arity** (the number of factors in the direct product) – again we can permit a more than denumerably infinite supply
- Connectives $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- Quantifiers $\forall, \exists$ and brackets [,]
- **context free languages** are given by a set of **nonterminals** V, a set of **terminals** Σ, and a **grammar** with **productions** in the form $A \rightarrow \alpha$ where $\alpha \in (\Sigma \cup V)^*$
- The **yield** of a CFG is the set of strings that can be obtained from a distinguished **start symbol** $S \in V$ by application of productions until no nonterminal is left

FOL (almost) by CFG

- Ignoring cardinality issues, the nonterminals include WFF, AF, Const, Var, and Rel_n. We will also have some technical symbols , (comma) and _ (placeholder). Brackets, parentheses, connectives and quantifiers are considered terminal symbols, as are the individual constants, variables, and relation symbols
- The rules for **Atomic Formulas**: $\mathrm{AF} \rightarrow R_n((\_,)^{n-1}\_)$ 'Each n-ary relation symbol must be followed by (, a string of n empty slots separated by n-1 commas, and terminated by)'
- $\_ \rightarrow c$, $\_ \rightarrow v$ 'Slots of n-ary relational symbols must be filled by constants or variables. So $R(a, x, b)$ is an Atomic Formula, but $S(x, \_)$ is an incomplete atomic formula (doesn't count in the yield, because it still has a nonterminal _)
- To check if a string is an Atomic Formula, you need to check if it starts with a relational symbol, what is the arity of that symbol, and whether the slots are filled by variables/constants

Moving from atomic to more complex formulas

1. WFF $\rightarrow$ [AF] ‘bracketing an atomic formula gives a well-formed formula’
2. WFF $\rightarrow$ [$\neg$ WFF]||[WFF $\vee$ WFF]||[WFF $\wedge$ WFF]||[WFF $\Rightarrow$ WFF]||[WFF $\Leftrightarrow$ WFF] ‘logical operations on WFFs lead to WFFs’
3. WFF $\rightarrow$ [($\forall$ Var) WFF]||[($\exists$ Var) WFF] ‘quantification’
4. We need to make sure that e.g. $[(\forall x)[(\exists x)R(a, x)]]$ is *not* a WFF ‘capturing variables’. This can’t be done with a CFG (Type 2) , but very easy with a linear bounded TM (Type 1)
5. HW7.1-3 Write ZFC1,2,5 in FOL Remember $=$ and $\in$ are binary relations