A matematika alapjai, 5. előadás

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Bináris relációk főbb tulajdonságai

- For any relation $R \subset (X \times Y)$ we can produce the *reversal* R^T and the *complement* R^C of R
- These are not the same! For equality $=^T$ is $=$ but $=^C$ is $\neq$
- HW4.2 Write formulas for R^T and R^C!
- A relation is **reflexive** iff $\forall a \in X : aRa$
- A relation is **irreflexive** iff $\nexists a \in X : aRa$
- A relation is **symmetric** iff $\forall ab : aRb \Rightarrow bRa$
- A relation is **antisymmetric** iff $\forall ab : aRb \wedge bRa \Rightarrow a = b$
- A relation is **transitive** iff $\forall abc : aRb \wedge bRc \Rightarrow aRc$
- Relations are sets: we can do union, intersection, complementation (relative to universe $X \times Y$)
- Composition of relations can also be defined provided types match: $R \subset X \times Y, S \subset Y \times Z$. We say $a(S \circ R)c \Leftrightarrow \exists b : aRb \wedge bSc$

Bináris relációk főbb típusai

1. Relations that enjoy the reflexive, symmetrical, and transitive properties are called **equivalence relations**
2. Relations that are reflexive, antisymmetric, and transitive are called **ordering relations**
3. Equivalence relations are covered in Chapter 9 of CPZ
4. They are closely related to partitions: every e.r. corresponds to a partition
5. We discuss CPZ 9.1-3 in class
6. HW4.3 is CPZ9.4
7. Divisibility (CPZ9.6). $a|b \Leftrightarrow b/a \in \mathbb{N}$
8. We do this now for natural numbers $\{1, 2, \ldots\}$, but it extends smoothly to integers $\mathbb{Z}$
9. HW4.4– is CPZ9.8,10,12,14,16,18,20,22

Partíciók

- A *partition* of a set A is a family of sets A_α such that for any $\alpha \neq \beta$ we have $A_\alpha \cap A_\beta = \emptyset$ and $\bigcup_\alpha A_\alpha = A$. By definition, we never consider the empty set a part of any partitition, so in the definition we may write "a family of *nonempty* sets"
- The partition can be finite (e.g. the sets of *even* and *odd* numbers partition the set of integers) or infinite
- Partitions are related to *equivalence relations*.
- There are two *trivial* partitions, when all elements are in the same set, and when all go in their own set

A függvények mint relációk

- All functions are relations, but not all relations are functions!
- The big difference is that functions have *unique output*, a relation F will be called a function only if $aFb \wedge aFc \Rightarrow b = c$
- Definitions of domain, codomain, range, and composition are the same. We don't write 30° cos $\sqrt{3}/2$, we write $\cos 30^\circ = \sqrt{3}/2$
- CPZ devotes Chapter 10 to functions, we will cover this in class today, but the entire chapter is homework to read. Exercises similar to those in CPZ Ch 1, 9, and 10 will be on the midterm
- Composition of functions is just like composition of relations: if $f : A \rightarrow B$ and $g : B \rightarrow C$ then $g \circ f : A \rightarrow C$
- Sometimes (often) more lax terminology is used, permitting functions to be defined only on a subset of their domain. For example, most people will talk about $\sqrt{\ }$ as an $\mathbb{R} \rightarrow \mathbb{R}$ function, even though its *natural domain* is only $\mathbb{R}_0^+$
- Other tricky point about $\sqrt{\ }$ is that output (depending on definition) is not unique

A legfontosabb függvény-tulajdonságok

1. Injective: different x-es map on different y-s: $f(x) = f(y) \Rightarrow x = y$
2. Surjective: codomain = range (codomain $\supset$ range is true by definition)
3. Bijective: both injective and surjective
4. **Theorem:** a function f is *invertible* $\Leftrightarrow$ it is *bijective*
5. **Proof:** We need to prove both $\Rightarrow$ and $\Leftarrow$. For $\Rightarrow$ we need to *verify* that the bijective properties follow from invertibility. For $\Leftarrow$ we will construct the inverse of a bijective function.
6. ($\Rightarrow$) What do we suppose? What do we need to prove?
7. ($\Leftarrow$) What do we suppose? What do we need to prove?
8. Discussion of CPZ Ex 10.18