

# UNIFYING FORMULAIC, GEOMETRIC, AND ALGEBRAIC THEORIES OF SEMANTICS

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# OUTLINE

1 BACKGROUND

2 SCHEMAS

3 SPACE

4 TIME

5 INDEXICALS

# INTRO

- In Lecture 3 we discussed the ‘up to quadratic, but not to cubic’ version of Smolensky’s tensorial program
- In practical terms, this means we use  $L$ , the linguistic subspace, spanned by word vectors, and  $L^2$ , of which we think in terms of perturbations on the scalar product obtaining at time  $t$
- These updates follow sentence/text comprehension, taking place maybe 2-3 times a second, as opposed to the centisecond timescale of speech recognition and the microsecond changes in the thought vector.
- We are dealing with the *naive* theory of space and time. There is **no** claim that this is somehow superior to the sophisticated theory (the Einstein field equations). The grand program of finding the ultimate truth about Space, Time, Causality, and other weighty matters by reverse-engineering the lexica/grammars of natural languages is **not** supported. Our slogan is *more morphology, less cognitive mysticism*

# MOTIVATION

- Spatial relations are often viewed as central to the system of cases that link predicates to their arguments Anderson, 2006
- In systems of knowledge representation, it is superbly tempting to have *WHERE* and *WHEN* links
- In fact the Navya-Nyāya system of Indian logic does this, using *pakṣa* for spatiotemporal locus
- Relation between temporal and nominal long noted (Partee, 1984)
- Preliminary to fuller formalization of Talmy, 1983; Talmy, 1988; Talmy, 2000 and much other work in cognitive semantics

# SCHEMAS

- Schemas, sometimes called scripts, are often used in GOFAI  
Minsky, 1975; Schank and Abelson, 1977 and go back to 1930s  
psychology
- Our first example will be the well known  
`commercial_transaction` schema
- This has four participants: *a buyer, a seller, some goods* and  
*money* or some other 'thing of value'
- Before the transaction, seller has the goods and buyer has the  
money, afterwards buyer has the goods and seller has the money
- The same schema is applicable both for the word *buy* and for *sell*

# THE EXCHANGE SCHEMA AS A VORONOID

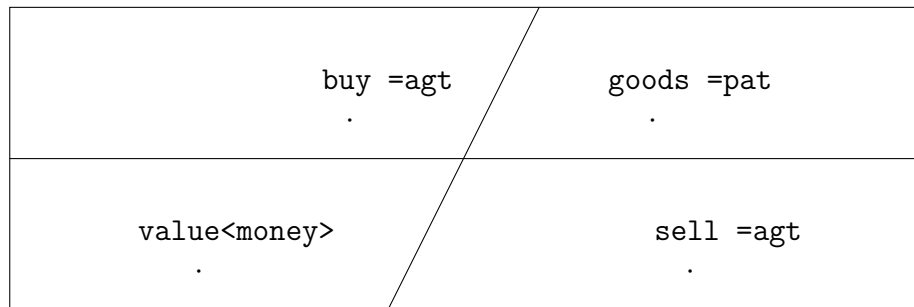


FIGURE: exchange\_

(Temporal aspects will be added shortly)

## AN INSTANCE

*And Jacob sod pottage: and Esau came from the field, and he was faint. And Esau said to Jacob, Feed me, I pray thee, with that same red pottage; for I am faint. And Jacob said, Sell me this day thy birthright. And Esau said, Behold, I am at the point to die: and what profit shall this birthright do to me? And Jacob said, Swear to me this day; and he sware unto him: and he sold his birthright unto Jacob. Then Jacob gave Esau bread and pottage of lentiles; and he did eat and drink, and rose up, and went his way*

Esau is asking for food, and Jacob asks Esau to *sell* his birthright. Esau was only asking for food, and it is Jacob who invokes the exchange schema, with the slots *seller* filled by Esau; *buyer* by Jacob; and *goods* by the birthright. Subsequently the schema is ratified by Esau swearing to it, and fulfilled by his eating the bowl of lentils. That this food is the 'thing of value' is unquestionable, but how does the vectorial semantics reflect this?



# PATTERN RECOGNITION

The four vectors  $\{\vec{v}(\text{buyer}), \vec{v}(\text{seller}), \vec{v}(\text{goods}), \vec{v}(\text{money})\}$  are the defining elements of the exchange schema as a set (the curly braces emphasize that their order is immaterial). Together, they define a polytope, the intersection of the positive half-spaces. The other 4 vectors in the example,  $\vec{v}(\text{Jacob}), \vec{v}(\text{Esau}), \vec{v}(\text{birthright})$  and  $\vec{v}(\text{food})$  are just points (or small polytopes) in  $L$ . What we are looking for is an equaliser  $Q$  such that after applying  $Q$  to the representation space  $R$  that reflects the state of affairs before Jacob making the offer, we obtain  $R'$ , where not just

- (1)  $\vec{v}(\text{buyer}) = \vec{v}(\text{Jacob})$  and
- (2)  $\vec{v}(\text{seller}) = \vec{v}(\text{Esau})$  but also
- (3)  $\vec{v}(\text{goods}) = \vec{v}(\text{birthright})$  and
- (4)  $\vec{v}(\text{thing\_of\_value}) = \vec{v}(\text{bowl\_of\_lentils})$  holds.

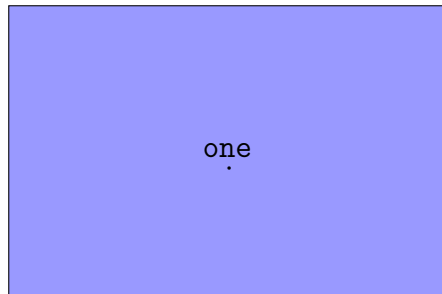
## PATTERN RECOGNITION CONT'D

These equations are created by several different mechanisms. Equations (1-2) come from resolving pronouns corresponding to speaker and hearer: the sentence *Sell me (this day) your birthright* is addressed to Esau, making him the seller, and spoken by Jacob, making him the buyer. Since the birthright appears in the patient slot of this sentence, we obtain the third equation by a general syntactic mechanism (whatever follows the verb is the object). The last equation is supported by the mechanism of pragmatic inference discussed in [S19:5.7](#): we know from earlier sentences that the *food* and the *bowl of lentils* are the same, we know Esau is faint, and he himself acknowledges that at this point the food is more important to him than his birthright: *Behold, I am at the point to die: and what profit shall this birthright do to me?* This establishes, from the seller's perspective, that the thing of value to be received for the goods is indeed the food.

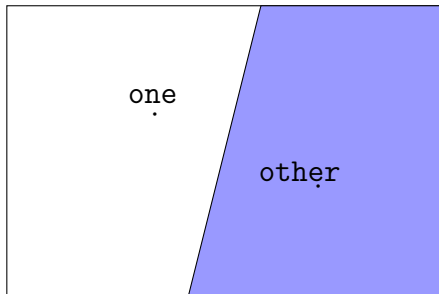
# EQUALITY, NOT JUST `is_a`

- We could use the scalar product mechanism we used for *be* to establish that Esau is in the seller halfspace, but we want more, we want to say that Esau is *the* seller of this instance of the schema
- This is why use equalizers (linear transformations that have e.g.  $\vec{v}(\text{seller}) - \vec{v}(\text{Esau})$  in the kernel) to the entire representation space  $L^2$ .
- Equalizers are used instead of variable binding both for pronouns and for other indexicals
- We will talk about *coercion* for what Fauconnier, 1985 calls 'projection mapping' to emphasize the Procrustean aspect of the process
- Heider-Simmel test see discussion in Gordon and Hobbs, 2017

# THE SIMPLEST SCHEMAS



one/all

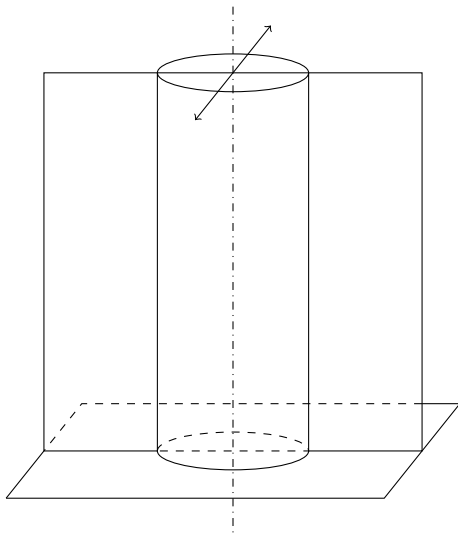


other

# THE PLACE\_SCHEMA

ground ·	inside body ·	about near ·	far ·
underside ·	front face ·	over top ·	

# EGOCENTRIC COORDINATES



# THE NAIVE THEORY OF SPACE

- Has “body” at the origin. When we interpret *John (is) at (the) office* what we do is coerce *office* on the body of the schema, and place John inside this body
- Body schema Head and Holmes, 1911 automatically mapped on gravity vertical (part of  $\Psi$ , since it is directly sensed by the inner ear)
- Mapping on cardinal directions otherwise culture-specific: in Sanskrit front is East *pūrva-*, right is South *dakṣina*, etc. whereas in Finnish *etelä* ‘South’ is from *ete-* ‘front’ and *pohjoinen* ‘North’ is from *pohja-* ‘bottom’
- about or near seems to be defined by extension of arms

# SOME DEFINITIONS RELATED TO GRAVITY

## VERTICAL

up	fel	sursum	do_golry	763	A
	after(at position), vertical(position er_ gen)				
down	le	deorsum	w_do111	1498	D
	vertical(gen er_)				
vertical	fu2ggo3leges	verticalis	pionowy	869	N
	direction, has top, has middle, has bottom, earth pull in direction				
fall	zuhan	cado	spadac1	2694	U
	move, after(down)				
stand	a111	sto	stac1	74	U
	=agt[vertical], =agt on two(foot)				



# METAPHORS WE LIVE BY

- 99% of metaphors go back to application of some schema
- The metaphor appeals to core meanings, not to some 'extension': when we say "top of the hill" this is regular usage both for *top* and for *hill*
- *acid* in *vinegar is an acid* means the exact same thing as in *an unnecessarily acid remark*
- If we don't get the effect immediately, we stipulate it (see heart below)
- Inspiration from Lakoff and Johnson, 1980; Lakoff, 1987; Fauconnier, 1985 and many others.

# HOW LANGUAGE FITS THE BODY SCHEMA

chin	aill	mentum	broda	73	N	
	part_of	face,	at/2744	centre,	under	mouth
face	arc	vultus	twarz	177	N	
	organ,	surface,	front,	part_of	head,	forehead part_of
	chin	part_of,	ear	part_of,	jaw	part_of
forehead	homlok	frons	czollo	1077	N	
	part_of	face,	front,	eye	under,	hair at, at temple,
front	elej	pars_prior	przo1d	608	N	
	part,	first				
left	bal	laevus	lewy	222	N	
	side,	has	heart			
right	jobb	dextra	prawy	1199	N	
	side,	lack	heart			
	organ,	joint,	at	hand,	at	end, arm has end
heart	szilv	cor	serce	2210	N	
	organ,	cause_[blood[move]],	love	in/2758,	centre	

# BOUND

- Another spatial schema of great importance, this time with only two participants: the distance, area, or volume that is being bound, which we will call `volume_` irrespective of dimension, and a `boundary_` which typically has one less dimension, e.g. a distance (line, one dimension) is bounded by points (zero dimensional).
- Only slightly related to the `place` schema: we can compare the `volume_` to the `body_`, but really the 'skin' that bounds the body is derived from the `boundary_` and not the other way around.
- In a *bound* statement we don't particularly identify with the spatial viewpoint of the `volume_` or the `boundary_`. Rather, the observer is floating somewhere, does not matter where.
- Central instances: `between`, `distance`

# THE CALENDAR SYSTEM

- The mathematical tool of choice: cyclic groups:  $C_{60}$  for seconds and minutes,  $C_{24}$  for hours,  $C_7$  for weeks, and so on.
- Calendars are culture- and language-dependent, our interest is with universal semantics
- We recognize  $C_2$  in frequentatives like H *-gat/get*
- There is also a particular 'Gnostic' world-view (remember the one/other schema): there are only now, and not-now, some other time. Sometimes it's light, sometimes dark, sometimes raining, sometimes not, sometimes we hunger, sometimes we are full.

# THE TENSE SYSTEM

- We recognize as many stages as there are distinct tenses.
- Typically past/present/future, but many languages subdivide the past (e.g. historic, remote, and recent past tenses) and/or the future. Maximum granularity seems to be 7.
- We recognize  $C_2$  in frequentatives like H *-gat/get*
- There is also a particular 'Gnostic' world-view (remember the one/other schema): there are only now, and not-now, some other time. Sometimes it's light, sometimes dark, sometimes raining, sometimes not, sometimes we hunger, sometimes we are full.

# VECTOR SEMANTICS OF TENSE

- No cyclic aspects, except for Ecc 1.9 “the thing that has been, it is that which shall be”
- We make as many copies of the representation space  $V = L \times L$  as there are tenses. For the standard past/present/future distinction this means three copies  $V_b, V_n, V_a$  ‘the world *before* the event, *now*, and *after*’
- But wait, isn’t this messing up the quadratic restriction on the Smolensky program? No, because the largest information objects are still  $cd^2$  size, where  $c \leq 7$
- We use  $V_n$  by default, and use the predicates *before* and *after* to refer to the other two. Think of these as three snapshots from a movie depicting some action
- Example: in the definition of `sell` we use `before(=agt has goods)`, `after(=agt has money)`

## TENSE VERSUS TIME

- In this (naive) model, time is discrete, rolls on automatically, the previous  $V_n$  becomes  $V_b$ , and  $V_a$  becomes  $V_n$ . Each of these worlds is timeless, and the time spent between them is underspecified.
- We only need to list changes: e.g. *former* means something was true in  $V_b$  but is no longer true in  $V_n$
- There is no cyclicity (outside Ecclesiastes). Consider *latinate ana-* as in *anabaptism* 're-baptising'; *anabiosis* 'return to life'; *anaclasis* 'reflect, turn back'; *anacrusis* 'pushing back'; *anadiplosis* 'repetition of a prominent word'; *anaphoric* 'repetition of a word'; *anaphylaxis* 'severe reaction to second or later administration of a substance'; *anatexis* 'melting again'; *anatocism* 'the taking of compound interest'
- Doubling back, returning, does not take us to the original concept, it takes us to the concept *again*, with some temporal marker or counter updated

# THE STATE OF THE ART

- Since the Message Understanding Conferences (Grishman and Sundheim, 1996) special attention is paid to the extraction of *numerical expressions* (NUMEX) such as monetary sums and dates.
- The notion of calendar dates has been extended to cover more complex time expressions (TIMEX)
- There is a standard Semantic Web representation schema, ISO TimeML associated to instances, intervals, etc. This grew out of earlier work on providing semantics for time expressions (Pustejovsky et al., 2003; Hobbs and Pan, 2004)
- Extracting this information from (English) text is difficult (Chang and Manning, 2012) and the parsing and normalization of time/date expressions is still an active research area (Laparra et al., 2018).



# LIMITATIONS OF TIMEML

*“In natural language, a very important class of temporal expressions are inherently vague. Included in this category are such terms as soon, recently, late, and a little while. These require an underlying theory of vagueness, and in any case are probably not immediately critical for the Semantic Web. (This area will be postponed for a little while).”*

The quote is from Hobbs (2004), the problem is still unresolved. TimeML is the weakest where the naive model is the strongest

# LIMITATIONS OF THE NAIVE MODEL

The naive model is the weakest where TimeML is the strongest, working with real numbers (and absolute geocoordinates). We don't have standard real numbers, we don't even support integer arithmetic (not even Robinson's  $Q$ , let alone  $PA$ ). Instead, we have the principle of non-counting, originating with Chomsky, 1965:

*For any natural language  $N$ , if  $\alpha p^n \beta \in N$  for  $n > 4$ ,  $\alpha p^{n+1} \beta \in N$  and has the same meaning*

To distinguish *great-great-great-great-great-great-great-grandfather* and *great-great-great-great-great-grandfather* we need to count on our fingers! Our approach is to do the work outside by means of a separate equation solver. This is also the approach taken in modern systems aiming at word problems such as Kushman et al., 2014, which uses Maxima

# INDEXICALS

- Personal pronouns are anchored to concepts such as *speaker* and *hearer* which are trivially accessible as parts of  $\Psi$  but do not project a static component on  $L$ .
- Spatial indexicals like *here*, *near*, *far* are accessed through the place schema. Pointing gestures are accessible through the sensory portion of the thought vector  $\Psi$
- We use coercion (equalizers) rather than `is_a` (subsetting)
- `now` is defined by the state of the suprachiasmatic nucleus see <https://www.nobelprize.org/prizes/medicine/2017/press-release> – as clear an example of embedded cognition as any

# BOBBERS

- We follow Meinong, as reconstructed in Parsons, 1974; Parsons, 1980, assuming words to be capable of denoting objects about which we only have partial information, partial even to the extent their very existence and identity are uncertain
- The denotations are similar to the *pegs* of Landman, 1986, we call them *bobbers*, partially defined individuals already tied to some properties that can be effortlessly computed from regions of  $\Psi$  that can lie outside the linguistic subspace  $L$ . When the water level rises, the bobber rises with it, and so does the bait linked to it by a fixed length of string
- For now the length of the string is zero: as our internal clock moves, so does now
- We define today as day, now and yesterday as day, after(today)

# MAIN TAKEAWAYS

- Theory of space based on highly simplified human body schema  
– no legs, arms, or anatomical detail
- We also have a notion of bound ‘something between something’ where the patient (the boundary) is a collective noun(phrase)
- Theory of time is based on tense systems (and will vary from language to language). We have as many “worlds” as there are tense-accessible time slices
- Pronouns/indexicals are bobbers, attached to the thought vector loosely

# Thank you!

Lecture and supporting materials available at  
<http://kornai.com/2021/ESSLLI>

Tomorrow: (choice of) negation, modality, probability, implicature  
Possible reading: Kornai *Vector Semantics* book draft  
<https://kornai.com/Drafts/advsem.pdf> Chapter 3

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