2nd Midterm

Foundations of Mathematics 2nd midterm

2020 Dec 1

M2.1 Let $S = \{1, 2, \ldots, 6\}$ and let $P(A) : A \cap \{2, 4, 6\} = \emptyset$ and $Q(A) : A \neq \emptyset$ be open sentences over the domain 2^S.
(a) Determine all $A \in 2^S$ for which $P(A) \wedge Q(A)$ is true
(b) Determine all $A \in 2^S$ for which $P(A) \vee \neg Q(A)$ is true
(c) Determine all $A \in 2^S$ for which $(\neg P(A)) \wedge (\neg Q(A))$ is true

M2.2 Determine the truth value of each of the following quantified statements:
(a) $\exists x \in \mathbb{R}, x^3 + 2 = 0.$
(b) $\forall n \in \mathbb{N}, 2 \geq 3 - n$
(c) $\forall x \in \mathbb{R}, |x| = x$
(d) $\exists x \in \mathbb{Q}, x^4 - 4 = 0$
(e) $\exists x, y \in \mathbb{R}, x + y = \pi$
(f) $\forall x, y \in \mathbb{R}, x + y = \sqrt{x^2 + y^2}$

M2.3 Let $S = \{8, 12, 20, 24\}$. Prove that if $\{x, y\}$ is a 2-element subset of S, then either $x + y = 8k$ for some even integer k or $x + y = 4l$ for some odd integer l.

M2.4 Prove or disprove the following:
(a) If A is an uncountable set, then $|A| = |\mathbb{R}|$
(b) There exists a bijective function $f : \mathbb{Q} \to \mathbb{R}$
(c) If A, B and C are sets such that $A \subseteq B \subseteq C$ and A and C are denumerable, then B is denumerable
(d) The set $S = \{\sqrt{2}/n : n \in \mathbb{N}\}$ is denumerable
(e) There exists a denumerable subset of the set of irrational numbers
(f) Every infinite set is a subset of some denumerable set
(g) If A and B are sets with the property that there exists an injective function $f : A \to B$, then $|A| = |B|$