About this course

- This is a first (foundational) course, there are no prerequisites
- The goal is to learn how mathematics is done: what are axioms, what are statements (lemmas, theorems) and how we prove these
- Students will learn about 'foundational' subjects: set theory, logic, category theory, algorithms – mostly by solving problems
- Students will also learn some practical skills such as LaTeX, bibtex, etc.
- Class webpage at https://kornai.com/2020/FoundationsOfMathematics
- Textbook ChartrandPolimeniZhang.pdf available from this page

Grading

- Prelim test (today) gives 0% of the grade!
- Weekly problem sets give 50% of the grade
- Class activity counts for 25%
- Final exam gives remainder of the grade (25%)

Material covered (not in this order)

- Intro set theory: sets, functions, relations, operations
- Intro logic: formulas, axioms, deduction, models
- Intro category theory: objects, arrows, functors, natural transformations
- Intro algoriths: input, output, computation
- Numbers: $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$
- Structures: groups, rings, fields, modules
- Probability: elementary foundations

Why is it like this?

1. Historically, math grew out of two main threads: counting and geometry
2. The foundations were initially Euclid's reaction to the Sophists (the same axiomatic method was used by Pāṇini to create the foundations of grammar)
3. Modern foundations were developed $\sim$ 1850-1950

1. Three subsets A, B, and C of $\{1,2,3,4,5\}$ have the same cardinality. Furthermore,

A 1 belongs to A and B but not to C

B 2 belongs to A and C but not to B

C 3 belongs to A and exactly one of B and C

D 4 belongs to an even number of A, B, and C

E 5 belongs to an odd number of A, B, and C

F The sums of the elements in two of the sets A, B, and C differ by 1

What is B?

2. (a) Compute $\cos(20^\circ)\cos(40^\circ)\cos(80^\circ)$
(b) Prove that the result is exact