Algebra Section I Summary

The course material is divided in five sections. So far, we covered only the first of these in its entirety:

> Integer arithmetic: divisibility, remainders, greatest common divisor, Euclidean algorithm, irreducible and prime numbers, fundamental theorem of arithmetic. Linear Diophantine equations, modular arithmetic, full and reduced systems of residues, solving linear congruences.

All of this material is covered in Chapters 1, 3, and 4 of Vinogradov – the clickable links to wikipedia given above are in general more detailed, and cover more material than what you will need. Also, Vinogradov has a great deal more material than we will require for the exam (for example §4 of Chapter 1, or §4 of Chapter 4 are not required). The problems at the end of each chapter are quite demanding, it is sufficient to restrict attention to the numerical exercises at the end of these chapters. (Much the same material is covered in Judson Chapters 2 and 3, see below.)

We also covered some additional material, not listed explicitly above, but logically very much part of it. This includes: Sets and operations of sets: union, intersection, complement relative to a universe. Subsets, elements of sets, empty set. Ordered pairs and n-tuples, Cartesian product, binary relations. Properties of relations: symmetrical, antisymmetrical, transitive, reflexive. Equivalence relations and partial orderings. Correspondence between partions of a set and equivalence relations on it. The Calculus class also covered functions, domain, codomain, injective, surjective, bijective.

All the set theory material is covered in Chapter 1 of Judson – again, the wikipedia pages made clickable here are in general more detailed, and cover more material than what you will need. In class we covered additional material that will *not* be required in the exam, such as the Peano axioms. Here only the last axiom, called in Judson *The First Principle of Mathematical Induction* on page 24, will be required, but the way we built integers $\mathbb{Z}$ from natural numbers $\mathbb{N}$ is not required. Also, Judson often uses matrix examples, and we will cover matrixes only in section three of the course. On the other hand, Judson's exercises are much easier, roughly at the level we require in tests, and spending time on solving these is strongly advised.

Another resource of great utility is the Shklarsky-Chentzov-Yaglom problem book. These problems, originally intended for exceptional high-school students, become considerably easier once you study modular arithmetic, and working on Chapters 2-4 is highly recommended, especially as these are often easier than the corresponding problems in Vinogradov.

We started to cover material from Section 2: Complex numbers, algebraic and trigonometric form, roots of unity. This is covered in Chapter 4.2 of Judson, we coverred Chapter 4.1 (cyclic groups) and the basics of group theory (Chapter 3) earlier.