Linguist and Philos
DOI 10.1007/s10988-013-9130-8

RESEARCH ARTICLE

Constraints on the lexicalization of logical operators

Roni Katzir · Raj Singh

Abstract We revisit a typological puzzle due to Horn (Doctoral Dissertation, UCLA, 1972) regarding the lexicalization of logical operators: in instantiations of the traditional square of opposition across categories and languages, the O corner, corresponding to 'nand' (= not and), 'nevery' (= not every), etc., is never lexicalized. We discuss Horn's proposal, which involves the interaction of two economy conditions, one that relies on scalar implicatures and one that relies on markedness. We observe that in order to express markedness and to account for a bigger typological puzzle, namely the absence of lexicalizations of 'XOR' (= exclusive or), 'all-or-none', and many other imaginable logical operators, one must restrict the basic lexicalizable elements to a small set of primitives. We suggest that an ordering based perspective, following Keenan and Faltz (Boolean semantics for natural language, 1985), makes the stipulated primitives that we arrive at more natural. We also propose a modification to Horn's proposal, based on recent work on implicatures, in which only the implicature condition is operative and in which markedness is part of the definition of the alternatives for scalar implicatures rather than an independent condition.

Keywords Logical operators · Negation · Lexicalization · Ordering · Scalar implicature · Contradiction · Markedness

R. Katzir (✉)
Department of Linguistics and Sagol School of Neuroscience, Tel Aviv University, Tel Aviv, Israel
e-mail: rkatzir@post.tau.ac.il

R. Singh
Institute of Cognitive Science, Carleton University, Ottawa, ON K1S 5B6, Canada
e-mail: raj_singh@carleton.ca

Published online: 23 May 2013

1 Introduction

Of the many potential denotations for the logical elements in natural language—connectives, quantifiers, modal operators, etc.—only a small subset is attested. This typologically puzzling state of affairs has led early work on binary connectives (in particular, Gazdar and Pullum 1976 and Gazdar 1979) and on generalized quantifiers (in particular, Barwise and Cooper 1981; Higginbotham and May 1981; Keenan and Stavi 1986; and van Benthem 1984) to offer a formal characterization of the subspace of operators that are realized, and then to attempt to derive this characterization from more basic principles.

Here we focus on one such puzzle, due to Horn (1972), which applies across several categories of logical elements. Horn frames his puzzle in terms of the traditional square of opposition: the so-called *O* corner, corresponding to *nand* (= *not and*), *nevery* (= *not every*), *nalways* (= *not always*), etc., is not expressed as one word in natural languages, contrasting with the remaining three corners. Horn proposes to account for the puzzle using two pragmatically motivated economy conditions. One condition keeps lexicalized inventories small: an *O* corner element like the hypothetical *nand* is not lexicalized since it can be derived as a scalar implicature of *or*. The other condition uses markedness, cashed out in terms of negativity (*and* and *or* are considered positive and unmarked, while *nor* and *nand* are considered negative and marked), to ensure that it is indeed the *O* corner that is not lexicalized rather than other possible scalar implicatures. We review Horn's puzzle in Sect. 1.1 and his proposal in Sect. 1.2.

In Sect. 2 we point out that Horn's proposal addresses only one lexicalization gap in each category, leaving many other gaps, such as the absence of words for *if-and-only-if*, *all-or-none*, etc., unaccounted for. Rather than conclude that Horn's characterization of the problem generalizes from the wrong data, we suggest that the generalization is correct as stated but should be seen in the context of a larger typological pattern: within the relevant categories, logical operators that, informally speaking, are not on a square of opposition for any category cannot lexicalize in the first place; within those that are on a square of opposition, Horn's pragmatic account handles the remaining puzzle of the missing *O* corner. Stating this larger generalization amounts to restricting the lexicalizable denotations in the relevant categories to a small set of primitives from which more complex operators can be built compositionally, a move that follows Keenan and Stavi (1986)'s work on quantificational determiners.

In Sect. 3 we suggest that the stipulations of lexicalizable primitives are made more natural if they are stated not as arbitrary denotations within their domains but rather, following Keenan and Faltz (1985), as ordering-sensitive operators: conjunctive operators like *and* and *every* are interpreted as inf (= greatest lower bound), and disjunctive operators like *or* and *some* are interpreted as sup (= least upper bound).

In Sect. 4 we note a challenge to Horn's account concerning the notion of implicature that is used. We suggest a modification that reflects recent work on the role of contradiction in the computation of scalar implicatures. We show that this change makes it possible to incorporate markedness directly into the definition of the alternatives in the first condition and eliminate the second condition. This move addresses the challenge and simplifies the original account.

1.1 The missing O corner

Looking at the Aristotelian square of opposition, Horn (1972, Chap. 4) points out a systematic asymmetry in how the different corners are lexicalized.[1] Across a family of categories that, informally speaking, allow us to state entries for logical operators—this family, call it $\mathcal{H}$, includes connectives, quantificational determiners, and modal verbs (and possibly several other categories, such as spatial and temporal adverbials, though we will not discuss them here)—there are entries that, again informally speaking, can be placed on a square of opposition for that category: a universal affirmative (A), such as *every*; a particular affirmative (I), such as *some*; a universal negative (E), such as *no*; and a particular negative (O), such as *not every*.[2,3] But while the two positive corners—the universal affirmative A and the particular affirmative I—are lexicalized across languages and across categories in $\mathcal{H}$ within a language, the universal negative E is much less likely to lexicalize, and the particular negative O corner does not lexicalize.[4] A schematic version of the square is shown in (1), and a

[1] See also Horn (2011). Horn's pattern is also discussed in Löbner (1983), Hoeksema (1999), Jaspers (2005), and Seuren (2006). A related generalization is studied by Barwise and Cooper (1981, p. 186).

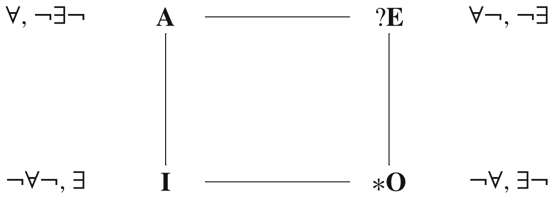
[2] The labels for the positive corners come from the Latin verb ***affirmo*** 'I affirm' and those for the negative corners come from ***nego*** 'I deny'. The traditional square of opposition is annotated with certain logical relations between its corners in the quantificational domain. See Kneale and Kneale (1962), Horn (1989), and Parsons (1997) for historical overview and discussion. Here we will ignore these relations and treat the square as a general schema—with concrete squares of opposition instantiated in various categories, as shown in (2)—that is useful for stating a particular pattern of lexicalization and markedness. For now we rely on an intuitive sense of what constitutes a square of opposition in a particular category. We make this sense precise in Sect. 2.

[3] We restrict our attention here to the categories in $\mathcal{H}$, and specifically to connectives, quantificational determiners, and modal verbs. These categories are commonly thought of as housing logical operators, and we will follow the literature in sometimes using terms that reflect this common perception. We wish to emphasize, however, that logicality plays no role whatsoever in our discussion and that for the purposes of this paper all that matters is the extensional definition of $\mathcal{H}$. That is, $\mathcal{H}$ can be thought of here simply as shorthand for *connectives, quantificational determiners, and modals*. As a reviewer points out, Horn (1972, 2011) proposes an extension of his account to cover lexicalization asymmetries in categories outside of $\mathcal{H}$, such as adjectives. See also Swanson (2010) for discussion of lexicalization in such categories (responding, in part, to an earlier version of the present proposal in Katzir and Singh 2009). We will not attempt to extend our discussion to such categories here.

[4] In the domain of connectives and quantificational determiners, the absence of O-corner lexicalizations seems absolute (as a reviewer points out, *not every* might seem like an exception but is not an actual determiner, as can be seen from its limited syntactic distribution). In the domain of modals, English provides the only currently known example of a potential O-corner lexicalization within $\mathcal{H}$ in the form of the contracted modal *needn't*. Horn (1972) notes this apparent exception but suggests a way in which it is compatible with his account of the typological pattern. As we will see shortly, Horn's account allows O-corner elements to be lexicalized in principle, relying on a potentially defeasible pragmatic mechanism to explain their apparent absence whenever the I and A corners are lexicalized. In the case of *needn't*, Horn attributes the apparent exception to the status of the modal *need* as an NPI. This idea can be spelled out by treating NPI modals as a category apart from non-NPI modals, one whose I corner is not lexicalized. As far as we can see, Horn's treatment of *needn't* carries over to the present discussion, but in what follows we will remain agnostic as to its precise status.

sample of lexical items in English corresponding to the different corners across several categories is shown in (2).[5]

(1)



(2)

	A	**I**	**E**	**O**
connectives	and	or	**n**either ...**n**or	***n**and
2	both	either	**n**either	***n**both
quant. determiners	every	some	**n**o	***n**every
temp. adverbs	always	sometimes	**n**ever	***n**always
modals	must	may	***n**may (ca**n't**)	***n**must (need**n't**)

Later work has provided further evidence for Horn's generalization. For example, von Fintel and Matthewson (2008) discuss an earlier survey of 33 languages by Matthewson in which all the languages surveyed had at least universal and existential quantifiers (that is, A- and I-corners) and suggest that this might indicate a language universal. The E corner has maintained its questionable status: there are many E-corner lexicalization gaps, both across categories and across languages; E-corner words often appear to be morphologically complex (note the *n*- and *-nt* marking in the English examples in (2)); and E-corner words often show scopal patterns, such as split-scope readings in Germanic, that have led to analyses in which the E element is decomposed into negation and an I-corner element.[6] To date, with the possible exception of *needn't*, no serious candidates for O lexicalizations in categories in $\mathcal{H}$ have been proposed.

1.2 Horn's proposal

Horn (1972) suggests that all the corners of the square—including the O corner—can be lexicalized in principle in any category in $\mathcal{H}$, and that his typological pattern is caused by pragmatic considerations: using an I-corner element in a sentence, write it $\phi[I]$, often gives rise to the implicature that $\phi[A]$ is false (where $\phi[A]$ is the same sentence but with the stronger A-corner element instead of the original I); this allows us (again, in certain cases and under certain assumptions) to convey $\phi[O]$ $(= \phi[\neg A])$

[5] In the domain of modals in English, negation can often be expressed with the ending *-nt*, as in the E-corner modals *can't* (or *cannot*), *mustn't*, and *shouldn't*. The morphological expression of this negation is somewhat different from the initial *n*- in the other rows in table (2), hence the parentheses.

[6] See Jacobs (1980), Sauerland (2000), Landman (2004), and Zeijlstra (2011), among others, for arguments supporting the decomposition of E-corner quantifiers in double negation languages into negation and an existential. In fact, Sauerland (2000) argues that there are no true E quantifiers in natural language (but see Geurts 1996, de Swart 2000, and Abels and Martí 2010 for non-decompositional accounts of split-scope phenomena).

simply by using $\phi[I]$. For example, if the speaker utters the sentence *John danced or Mary sang*, the hearer can sometimes conclude that the sentence *John danced and Mary sang* is false, an inference which, if we had the O element *nand*, we could express by saying *John danced nand Mary sang*. This, in turn, makes the O element somewhat redundant.[7,8]

To spell out this idea, Horn (1972) proposes two economy conditions on lexical inventories: one that keeps inventories small, and another that keeps them unmarked. The first condition, stated in (3), captures the idea that lexicalizing an operator z as part of an inventory X is superfluous—and hence bad—if z happens to be a scalar implicature of some other operator in X. The formulation in (3) looks at what an inventory X covers through membership or implicature, where coverage is defined as in (4) and written $[X]$, and implicature is discussed immediately below and written $\rightsquigarrow$. If two inventories cover the same elements but one is bigger than the other, the bigger one loses.

(3) Gricean Condition: Let X and Y be two inventories of logical operators such that $[X] = [Y]$. If $Y \subset X$, X cannot be lexicalized.

(4) Let X be an inventory of logical elements. For any logical element z, we will say that X *covers* z if (a) $z \in X$, or (b) there is some $y \in X$ such that $y \rightsquigarrow_X z$ (that is, X covers z if z is either a member of X or the scalar implicature of some member of X). We write $[X]$ for the set of all elements that are covered by X.

Implicatures are usually defined for whole sentences, but the present discussion will be facilitated by an implicature-like notion for individual operators.[9] We provide a first, intentionally simplistic attempt at such a notion in (5), which we will revise in Sect. 4 below. In (6)–(8) we provide additional definitions and notation following standard practice in generalizing from type t to higher types. The categories in $\mathcal{H}$, to

[7] There seems to be no cognitive restriction against processing the O corner—it is routinely derived as an implicature of I (Grice 1989). There also seems to be no cognitive restriction against learning an O lexical item—in the experiments reported in Hunter et al. (2009) and Hunter and Lidz (2012), children seem quite capable of learning a word like *nevery*. Moreover, if Horn is right in treating *needn't* as a (complex) O-corner lexicalized modal, this would be further evidence that the O corner is lexicalizable in principle. We believe that the apparent lack of cognitive restrictions on learning or processing O lends support to Horn's idea that the O corner is lexicalizable in principle and is blocked by pragmatic considerations.

[8] While the inspiration for Horn's proposal is pragmatic, he does not claim that pragmatic inferences make the O corner fully unnecessary. Indeed, there are many cases in which a speaker may wish to convey $\phi[O]$ and cannot use $\phi[I]$ to do so. Moreover, the way in which the partial redundancy of the O corner blocks the lexicalization of O need not be part of synchronic grammar. As pointed out to us by Paul Portner, Horn's account is in principle compatible with an implementation that uses diachronic change and language acquisition. These comments apply also to our implementation of Horn's proposal below.

Note that not just any informal sense of occasional redundancy translates into typological absence: *and* is stronger than *or*—*John danced and Mary sang* always conveys *John danced or Mary sang*—and yet the two operators co-exist as lexicalizations in many languages.

[9] The starting point for Horn's account is the observed inferences in sentences using elements from categories in $\mathcal{H}$. However, since the account is based on computing the implicatures of elements within hypothetical inventories, we will not be able to rely on observed inferences alone and will have to work with precise definitions. Whenever the inventory under discussion is lexicalized, we will of course expect the inferences predicted by the definitions to match those that are observed empirically.

which our discussion here is restricted, all end in t. We leave open the question of whether Horn's reasoning should be used for inventories in other types.

(5) OPERATOR- LEVEL SCALAR IMPLICATURE (first attempt, to be revised in (34)): Let Y be a set of operators. For any two operators y and z we say that z is an *operator-level scalar implicature* (OSI) of y given Y, written $y \rightsquigarrow_Y z$, *iff* both of the following conditions hold:

a. $\neg z \in Y$

b. $\neg z \sqsubset y$

(6) We will say that a type τ *ends in t* iff either $\tau = t$ or $\tau = \sigma_1\sigma_2$, where σ_1 is any type and σ_2 ends in t

(7) Let x be of a type τ that ends in t. The *negation* of x, written $\neg x$, is defined as (a) $1 - x$, if $\tau = t$, or (b) $\lambda f_{\sigma_1}.\neg x(f)$, if $\tau = \sigma_1\sigma_2$

(8) Let x and y be of a type τ that ends in t. We will say that x *entails* y, written $x \sqsubseteq y$ iff either (a) $\tau = t$ and $x \rightarrow y$, or $\tau = \sigma_1\sigma_2$ and for all z of type σ_1, $x(z) \sqsubseteq y(z)$. If $x \sqsubseteq y$ but it is not the case that $y \sqsubseteq x$ we will say that x *asymmetrically entails* y, written $x \sqsubset y$

(3) predicts that lexicalizing the full $X = \{A, I, E, O\}$ square in some domain will be bad by considering the smaller inventory $Y = \{A, I, E\}$ in the same domain: generally $I \rightsquigarrow_Y \neg A$, so lexicalizing O $(= \neg A)$ will be superfluous in the sense of (3), since $[X] = [Y]$ and $Y \subset X$. For example, in the domain of sentential connectives, we can rule out the lexicalization of the full hypothetical square $X = \{$*and*, *or*, *nor*, *nand*$\}$ by considering the smaller inventory $Y = \{$*and*, *or*, *nor*$\}$ and noticing that $[\{$*and*, *or*, *nor*, *nand*$\}] = [\{$*and*, *or*, *nor*$\}]$: *and*$\in Y$ and *and*$\sqsubset$*or*, so we obtain the OSI *or*$\rightsquigarrow_Y \neg$*and* (= *nand*). The inventory $\{A, I, E\}$ thus blocks the larger inventory $\{A, I, E, O\}$.[10] Note that the OSI *or*$\rightsquigarrow$ $\neg$*and* (= *nand*) also corresponds to the observed inference mentioned earlier from a sentence like *John danced or Mary sang* to $\neg$[*John danced and Mary sang*] (= *John danced nand Mary sang*).[11] While not strictly necessary for the system to work, the correspondence between OSI and observed inference is certainly part of the appeal: the same machinery used to derive observed patterns of scalar implicature can be re-used to account for the typological puzzle.

In the domain of quantificational determiners, it also seems empirically correct that I implicates $\neg A (= O)$, but deriving this fact is complicated by the possibility of

[10] Note that there is no two-element or one-element inventory that blocks $\{A, I, E\}$ by the same mechanism. On the other hand, a smaller inventory is not banned, either: a language is free to lexicalize an inventory with fewer than four elements in principle.

[11] To avoid clutter, we are using a certain amount of shorthand. For example, we will use a string like *and* either for the relevant syntactic element or for its denotation wherever the context makes the choice clear (where the context does not help, we will use $[\![\cdot]\!]$ for the denotation), sometimes marking the scope of negation using square brackets, as in $\neg$[*John danced and Mary sang*] (instead of $\neg[\![$*John danced and Mary sang*$]\!]$), and we write *John danced nand Mary sang* instead of $[\![$*John danced nand Mary sang*$]\!]$ in a variant of English in which the only change is the addition of a connective *nand* that has the semantics of $\neg[\![$*and*$]\!]$. We also talk derivatively about lexicalizing a whole inventory, by which we mean lexicalizing every element in that inventory (and without committing to the existence of a lexicon in its traditional sense). We hope that this does not give rise to confusion.

empty domains, which affects the relevant entailment relations. Thus, while there is an observed inference from *Some boy dances* to $\neg$[*Every boy dances*] (= *Nevery boy dances*), it is not the case, on standard assumptions, that *Every boy dances* logically entails *Some boy dances*, since it is possible that the set of boys is empty, in which case *Every boy dances* is true and *Some boy dances* is false. That is, *every* $\not\sqsubset$ *some*. Since the implicature seems to be present—and since Horn's proposal relies on it to block the O corner in the quantificational domain—the theory must be modified to account for it. A simple but potentially problematic option, which we will assume here, is that both sentence-level implicatures and OSIs take into account the existential import of A (that is, the inference that the domain is not empty). If there are boys and every boy dances, then it is also the case that some boy dances.[12]

The condition in (3) blocks the full four-corner inventory, but it does not choose a unique three-corner winner. In particular, it allows us to repeat the account just sketched but using the hypothetical three-corner inventory $Y' = \{A, E, O\}$ instead of the attested Y. To see this, note that generally in Y', $O \rightsquigarrow_{Y'} \neg E$, so lexicalizing $I (= \neg E)$ is superfluous in the sense of (3), since $[X] = [Y']$ and $Y' \subset X$. For example, in the domain of sentential connectives, the full hypothetical square $\{$*and*, *or*, *nor*, *nand*$\}$ is ruled out since *nor*$\in Y'$ and *nor*$\sqsubset$*nand*, so according to our definition *nand*$\rightsquigarrow_{Y'} \neg$*nor* (= *or*).[13] As far as (3) is concerned, then, a language can choose to cover the full square X by using the inventory Y'. Since Y' is empirically unattested, we need a way to restrict lexicalization further.

To eliminate the unattested Y' and choose the attested Y, Horn invokes a second condition that applies to the inventories that survive (3). This condition, stated in (9), captures the idea that, all things being equal, a marked inventory is dispreferred if a less marked inventory is available. Markedness, in turn, is cashed out in terms of negation: the more instances of negation among the elements of an inventory, the more marked it is. The E and O corners are taken to be negative and marked, each treated as having one instance of negation; the A and I corners are taken to be positive and unmarked.[14]

(9) Negation Condition: Let X and Y be two inventories such that $[X] = [Y]$. If X contains more instances of negation than Y, X cannot be lexicalized

In the example above, the attested $Y = \{A, I, E\}$ has one instance of negation, as part of E, while the unattested $Y' = \{A, E, O\}$ has two instances of negation, one in E and one in O. The attested inventory wins.

[12] As a reviewer points out, existential import has played an important role in discussions of the logical relations underlying the square of opposition. See, in particular, Strawson (1952). See Parsons (1997) for historical overview and critical discussion.

In Sect. 4 we will adopt the idea of negating not only stronger alternatives but also logically independent ones. This move will suggest a derivation of the relevant implicatures without recourse to existential import, but we will not pursue this possibility in this paper.

[13] Again, in the domain of quantifiers the requisite entailment relation does not hold due to the possibility of empty domains. Here, too, we can appeal to existential import for the purposes of the present discussion.

[14] See Horn (1989, especially Sect. 1.2 and chap. 3), for a cross-disciplinary survey of the markedness of negation and much relevant discussion.

The two remaining three-corner inventories, $\{A, I, O\}$ (where $O \rightsquigarrow \neg I(= E)$) and $\{I, E, O\}$ (where $I \rightsquigarrow \neg O(= A)$), do not block the full square. In both cases, the relevant entailment relation fails to hold (regardless of existential import), and the necessary implicature does not arise. This means that if a language tries to cover the full square in some domain, it will not be able to do so with $\{A, I, O\}$ or with $\{I, E, O\}$.[15]

2 Contextualizing Horn's pattern

2.1 Operators that are not on a square

Horn (1972)'s proposal blocks the lexicalization of logical operators using scalar implicatures. For example, *nand* is blocked using the negation of *and* when *or* is used. When we try to account for the absence of some unattested operator z but do not have lexicalized elements that can give rise to the relevant OSI, we are often left without an account of why the lexicalization of z does not occur. For example, while (3) allows us to eliminate *nand*, it does not help us eliminate the material biconditional $\leftrightarrow$, which is also never lexicalized. To see this, recall that Horn's first condition would allow us to block $\leftrightarrow$ in some inventory X only if we could present $\leftrightarrow$ as an OSI of some other element x within an inventory $Y \subset X$. But for that to happen, there must be some lexicalized $z \in Y$ for which $\leftrightarrow = \neg z$. This, however, is not the case: the negation of $\leftrightarrow$ is XOR (= exclusive or), another operator that is never lexicalized.[16] Other examples for connectives that are never lexicalized are the material implication $\rightarrow$ and the converse implication $\leftarrow$.[17] In fact, of the 16 possible binary bivalent connectives, the only ones that are ever lexicalized in natural languages seem to be *and*, *or*, and occasionally *nor*.[18]

In the domain of quantificational determiners, there is no lexicalized *every-or-no* (along with many other examples), again without an implicature to account for the gap.

[15] Note that $\{A, I, O\}$ and $\{I, E, O\}$ are both redundant in the sense of condition (3): $[\{A, I, O\}] = [\{A, I\}]$ and $[\{I, E, O\}] = [\{E, O\}]$. This means that neither will be lexicalized. On the other hand, neither $Y = \{A, I, E\}$ nor $Y' = \{A, E, O\}$ is redundant.

[16] In principle, it is possible within Horn's system that an operator will survive the first condition but will still not be lexicalized due to the second condition; this does not seem to be of help in the current case.

[17] See Lewis (1975), Kratzer (1981, 1986), and Heim (1982) for why *if* is not a plausible candidate for $\rightarrow$.

[18] One might try to account for the impossibility of XOR (and of *some-but-not-all* and *may-but-not-must*) by amending the original condition slightly so that it will capture XOR's close relation to the square. XOR itself is not an OSI of any corner of the square, but it is a possible *strengthened meaning* of the I corner: when the I corner is used and generates an $O(= \neg A)$ OSI, the result is a stronger reading, in which I is conjoined with its O OSI. For example, when *John dances or Mary sings* is used, the implicature *¬[John dances and Mary sings]*, when generated, means that the original sentence could be paraphrased as *John dances XOR Mary sings*. The close relation of XOR and *some-but-not-all* to the square has been recognized in the literature, where, following Blanché (1953, 1969), the strengthened meaning $I \wedge O$ is sometimes added to the diagram in (1) as the nadir and given the label Y. See Horn (1990, 2011), Béziau (2003), Moretti (2012), and references therein for discussion. As pointed out to us by a reviewer, a slight modification of Horn's account could thus analyze the Y corner as similar to the O corner in being lexicalizable in principle but blocked pragmatically. In our implementation, we could incorporate this idea by extending the definition of covering, in (4), to include strengthened meanings alongside OSIs. This move, however, does nothing to explain the absence of lexicalized $\rightarrow$, $\leftarrow$, and other connectives. We will set aside the question of whether strengthened meanings are also lexicalizable in principle, hoping that it can be settled independently of the rest of the discussion.

Indeed, with the exception of quantity and number words (which we turn to in Sect. 2.3), it is doubtful whether any quantificational determiner other than *every*, *some*, and occasionally *no* can lexicalize. A similar state of affairs holds in the domain of modal verbs.

It seems, then, that Horn's system can at best account for a small subpart of a more general puzzle. Across the categories in $\mathcal{H}$ there are entries that, informally speaking, can be placed on a square of opposition for that category: a universal affirmative (A), a particular affirmative (I), a universal negative (E), and a particular negative (O). Focusing on any specific square of this kind, the O corner is not lexicalized and Horn's proposal provides an explanation for this gap (assuming the A and I corners are lexicalized). For the unlexicalized entries of the same category that are not corners of such a square (or strengthened meanings thereof, as mentioned in note 18), Horn's proposal offers no help.

An obvious, if somewhat extreme solution is to augment Horn's account with a separate principle that applies within the categories in $\mathcal{H}$. In those categories, the principle prevents entries that do not belong on a square of opposition from lexicalizing in the first place, thus allowing us to focus on the squares of opposition for the relevant categories and on their missing O corner. Here is a preliminary attempt:

(10) Constraint on lexicalization (preliminary version; to be reattempted in (17)): For every category $C \in \mathcal{H}$, both of the following hold.
   a. There is a unique square of opposition for C, with a universal affirmative (A), a particular affirmative (I), a universal negative (E), and a particular negative (O)
   b. The only members of C that are potentially lexicalizable are the corners of the square of opposition for C

The constraint in (10) will be of little use unless we can tell what constitutes a square of opposition in a particular category. An unattractive option would be to list the corners of the square in each of the categories in $\mathcal{H}$. Fortunately, this is not necessary. The squares of opposition in the different categories are closely related: the A corners always express some form of (generalized) conjunction; the I corners always express some form of (generalized) disjunction; and so on. We can use this fact to define the square by stipulating its corners in one category, for example for connectives, and then deriving the corners in the other categories by rule.

If we use connectives as the domain in which we list the stipulated denotations of the four corners of the square, we might consider listing the four truth tables for the four operators: A is a conjunction, which takes two bivalent arguments, x and y, and returns 1 if $x = y = 1$ and 0 otherwise; I returns 0 if $x = y = 0$ and 1 otherwise; E returns 1 if $x = y = 0$ and 0 otherwise; and O returns 0 if $x = y = 1$ and 0 otherwise.

Our stipulations should be somewhat more detailed than that, however. A first complication is that connectives can apply to more than two arguments, as in *John jumps, Bill sleeps, and Mary smokes*. A second complication is that connectives such as *and* and *or* can apply also to non-sentential constituents. For example, *John and Mary jump*, where the subject is coordinated, and *John jumps and smiles*, where the verb phrase is coordinated. Following Keenan and Faltz (1978, 1985), Gazdar (1980), and Partee and Rooth (1983), sub-sentential coordination is usually handled semantically, by manipulating the types of the arguments and by extending the connectives inductively to higher types. In *John and Mary jump*, for example, we turn the subject into a generalized quantifier: we first raise j ($=[\![John]\!]$) and m ($=[\![Mary]\!]$) to the Montagovian individuals $\lambda P_{et}.P(j)$ and $\lambda P_{et}.P(m)$ and then conjoin them using a generalized form of conjunction, which can be defined as in (11).[19]

(11) Let $x_1, \ldots, x_n$ be of a type τ that ends in t. The *conjunction* of $x_1, \ldots, x_n$, written $\sqcap(x_1, \ldots, x_n)$, is (a) $\wedge(x_1, \ldots, x_n)$ (that is, 1 *iff* $x_1 = \cdots = x_n = 1$; 0 otherwise), if $\tau = t$, and (b) $\lambda f_{\sigma_1}. \sqcap (x_1(f), \ldots, x_n(f))$ if $\tau = \sigma_1\sigma_2$

Using (11), we can summarize our stipulation of the square of opposition in the domain of connectives as follows (we will sometimes use the shorthand $\sqcup$ for $\neg\sqcap\neg$)[20]:

(12) Truth-tabular stipulation of the square of opposition for n-ary connectives:
a. $A = \lambda P \in D_{\tau}{}^{n}. \sqcap (P)$
b. $I = \lambda P \in D_{\tau}{}^{n}.\neg \sqcap \neg(P)$
c. $E = \lambda P \in D_{\tau}{}^{n}. \sqcap \neg(P)$
d. $O = \lambda P \in D_{\tau}{}^{n}.\neg \sqcap (P)$

Definition (12) provides the content required for principle (10) in the domain of connectives: what can be lexicalized in this domain is not some abstract notion of square corners but the four concrete entries in (12).

Moving on to quantificational determiners, it has often been noted that *every* is a generalized form of conjunction—one where the conjuncts are not provided explicitly but rather by description, and their number is not bounded.[21,22] For example, *Every boy jumps* is true *iff* Adam jumps, and Bill jumps, and Caleb jumps, and …, where {Adam, Bill, Caleb, …} is the (finite) set of all boys; similarly, *some* is a generalized form of disjunction. Equivalently, we can think of *every boy* as the conjunction of the Montagovian individuals corresponding to Adam, Bill, Caleb, …, and we can think

[19] We will not discuss the case of so-called non-boolean conjunction, as in *John and Mary met* and *John and Mary are a couple*.

[20] To simplify exposition, the entries for coordination here and below are schemas that assume an independent operation of *tuple-formation*. Negation, ¬, is defined as in (7) above but extended to tuples so that $\neg{<}x_1, \ldots, x_n{>} := {<}\neg x_1, \ldots, \neg x_n{>}$.

[21] See McCawley (1972) for an early discussion of this point.

[22] The idea that elements like *every* and *some* are quantificational determiners, taking a predicative argument (an NP of type et) and returning a generalized quantifier of type $<et, t>$, has been widely accepted following Barwise and Cooper (1981) but has come under closer scrutiny in recent years. In particular, Matthewson (2001) has argued based on evidence from St'át'imcets that quantificational elements like *every* take an individual of type e as their argument. This argument of type e is obtained from the NP through a separate determiner, which is obligatory in the case of argument noun phrases. (Matthewson (2011) suggests that the choice between this and the traditional determiner entry might be subject to variation both cross-linguistically and within a language.) A different deviation from Barwise and Cooper (1981)'s view is the choice-function approach argued for by Sauerland (1998, 2004) and Abels and Martí (2010), in which quantificational determiners leave behind a trace that is a choice function of type $<et, e>$ and are interpreted in a higher position. While such approaches require a modification of the standard entries, they maintain the conjunctive and disjunctive cores of the relevant operators, and as far as we can see they are compatible with the current discussion.

of *some boy* as the disjunction of the same Montagovian individuals. On this second formulation, *every boy* raises a ($=[\![Adam]\!]$), b ($=[\![Bill]\!]$), and all other individuals to which the predicate denoted by *boy* applies to $\lambda P_{et}.P(a)$, $\lambda P_{et}.P(b)$, etc., after which these Montagovian individuals are conjoined. If we were to extend conjunction to sets, we could say that we are conjoining the elements of the set $\{\lambda P_{et}.P(x) : [\![\text{boy}]\!](x)\}$. Definition (13) provides the relevant extension of coordination to sets, and definition (14) uses this extension to state the generalization of connectives to quantificational determiners.[23]

(13) Let τ be a type that ends in t, let X be a finite set of elements of type τ, and let $x_1, \ldots, x_n$ be some enumeration of the elements of X.

a. $\bigsqcap X := \sqcap(x_1, \ldots, x_n)$

b. $\bigsqcup X := \sqcup(x_1, \ldots, x_n)$

(14) Basic quantificational determiners:

a. $[\![\text{every}]\!] = \lambda f_{et}.\bigsqcap\{\lambda P_{et}.P(x) : f(x)\}$

b. $[\![\text{some}]\!] = \lambda f_{et}.\bigsqcup\{\lambda P_{et}.P(x) : f(x)\}$

As for intensional operators such as modal verbs, we can translate their quantificational force into generalized conjunction and disjunction as long as we are willing to assume (perhaps unrealistically) that their domains are also finite[24]:

(15) Modal verbs (where R is an appropriately selected accessibility relation):

a. $[\![\text{must}]\!]^w = \lambda p_{st}.\bigsqcap\{p(w') : wRw'\}$

b. $[\![\text{may}]\!]^w = \lambda p_{st}.\bigsqcup\{p(w') : wRw'\}$

We have thus limited our stipulations: instead of expressing (10) by listing the denotations of square corners across categories, we have provided one family of stipulations, stated in (12), and then extended them by rule to the domain of quantificational determiners and the domain of modal verbs. In all cases, an operator that is some combination of conjunction and negation can be viewed informally as the content core, which is embedded within a skeletal entry built with lambda abstraction, set formation, and similar operations.[25]

---

[23] These extensions to sets are well-defined since both $\sqcap$ and $\sqcup$ are invariant to repetitions and reorderings of their arguments.

[24] The entries for modal verbs in (15), like the entries for connectives and determiners above, are based on traditional semantic accounts of these elements. As pointed out to us by Paul Portner, other accounts have been proposed in the literature, and it is by no means clear that the present account can be restated in terms of those other approaches.

[25] Some of this skeletal structure is part of the various lexical entries, but it is also possible that some of it is created dynamically, via type-shifting operations and their like. The present discussion has little to say regarding the division of labor between the two. Providing a full characterization of the possible logical operators in natural language will of course require a precise characterization of the allowable skeletal operations. We hope, however, that this can be left for a separate occasion and that the current discussion can be based on the informal identification of the operators under discussion with the appropriate conjunctive or disjunctive primitive.

2.2 Positive and negative operators

The constraint on lexicalization in (10) will need to be modified further in order to take markedness into account. Recall that in order to work as described above, Horn's account needs A and I to be unmarked and positive and E and O to be marked and negative:

(16) Positive and negative corners of the square
  a. A and I are positive and unmarked
  b. E and O are negative and marked (each has one instance of negation)

This seems intuitive enough, and it correlates with observed markedness patterns across the categories in $\mathcal{H}$, as noted earlier: the E and O corners often or always do not lexicalize, and when the E corner does appear to be lexicalized, it is often marked morphologically (as with *n-* in English), and it sometimes splits into negation and an existential (as in the so-called split-scope readings in Germanic). In terms of truth tables of the kind used in (12), however, it is hard to see how this distinction might be captured. In what sense, for example, is the truth table for *and* basic and positive, and in what sense is the table for *nor* non-basic and negative?

A simple solution is to state that only the A and I corners can be lexicalized as simplex. The negative corners E and O must be derived structurally, by combining a node for $\neg$ (expressed as *n-* in English and having the denotation in (7) above) with a node for A or I: $E := \neg I$ and $O := \neg A$. We can now add a statement about the negative corners to our constraint on lexicalization[26]:

(17) Constraint on lexicalization (negation-aware): For every category $C \in \mathcal{H}$, both of the following hold.
  a. Basic case: The only simplex operators in C are A (the rule-based extension of $\bigsqcap$ in C) and I (the rule-based extension of $\bigsqcup$ in C)
  b. Marked case: For an operator μ defined as in (17a), it may be possible to lexicalize $\neg\mu$, and the result is marked

With (17) we have finally reached a context where only squares of opposition in the appropriate categories can be lexicalized in principle and where the pattern of negativeness and markedness is as required for Horn's account to work.[27] The approach we have chosen follows the lead of Keenan and Stavi (1986), who argued that in the

[26] Note that there are various factors that can prevent the combination $\neg\mu$ (where μ is generalized conjunction or disjunction) from lexicalizing at all. For example, Hoeksema (1999) suggests that negation can incorporate into a quantificational determiner in a given language only if the basic word order in that language makes $\neg$ and μ adjacent in the non-incorporated case.

[27] Could we get away with limiting the stipulation in (17) to just one positive lexicalization per category? The idea would be to stipulate just one basic positive operator in (17a)—either A or I—and derive the remaining corners through negation. As it turns out, this will fail to derive (16) regardless of which of the two corners is taken as basic. If A is the basic primitive, the other corners are $I := \neg A \neg$, $E := A\neg$, and $O := \neg A$. If I is the basic primitive, the other corners are $A := \neg I \neg$, $E := \neg I$, and $O := I\neg$. In both cases, E and O are marked by an instance of negation, which seems in line with the fact that these corners are indeed marked, as discussed earlier. However, with both choices of primitive, the other positive corner (I if A is primitive; A if I is primitive) is even more marked than either of the negative ones, with

domain of determiners, there is a small set of basic determiners from which the rest of the domain is built compositionally, adding structure in the process (see also Szabolcsi 2010, especially Sect. 12.5).

2.3 Other operators: the problem of quantity and number words

In the domains of connectives and modals, (17) seems to be correct as stated. In the domain of quantificational determiners, on the other hand, number words such as *five* and quantity words such as *many*, *few*, and *most*—we will refer to such elements collectively as *quantity and number words* (QNWs)—seem to pose counter-examples to (17). It is conceivable, of course, that (17) captures only a general tendency. While such a concession might ultimately be needed, we think that the literature has suggested interesting ways in which (17) can be correct as stated.

(17) says that the only simplex elements in any category $C \in \mathcal{H}$ are A (the rule-based extension of $\bigsqcap$ in C) and I (the rule-based extension of $\bigsqcup$ in C). In the domain of quantificational determiners, this means that the only simplex elements are the universal *every* and the existential *some*. Anything else, if (17) is correct, is either not a quantificational determiner (which, for the present discussion will mean that it is not of type $< et, < et, t >>$) or not simplex. We will review two approaches to QNWs from the literature. One approach, which we will review first, is committed to QNWs other than *most* having low-type, non-quantificational lexical entries. On this approach, any quantificational use of such elements is derived through the application of a type-shifting operation or through the insertion of a distinct quantificational element into the structure. For *most*, we will discuss recent arguments in favor of a decomposition into *many*, which on this first approach is an adjective and not a quantificational determiner, and the superlative morpheme *-est*. The other approach, reviewed next, sees all instances of QNWs as being quantificational determiners at some level but is still compatible with (17) via further decomposition.

The first approach, which, following Landman (2004) we will refer to as the *adjectival* view, assigns adjectival semantics—either of type et, as in the following entries, or their type $< et, et >$ variants—to the lexical entries of *few*, *many*, and *five*[28,29]:

(18) a. $[\![\text{five}]\!] = \lambda x_e . |x| \geq 5$

b. $[\![\text{few}]\!] = \lambda x_e . |x| \leq d$, for some contextually determined d

c. $[\![\text{many}]\!] = \lambda x_e . |x| \geq d$, for some contextually determined d

Footnote 27 continued
two instances of negation in each case. This goes directly against the empirical pattern of markedness: the positive corners lexicalize across languages and across categories, they do not bear any morphological marking that suggests negation, and they do not give rise to split-scope readings.

28 Our interest here is (17), which deals with simplex elements (or their combination with negation). Consequently, we focus on basic numerals and ignore complex numerals like *twenty-seven*, *five hundred*, etc. Complex numerals have recently been analyzed as (combinations of) lexical elements of a nominal or adjectival type rather than as quantifiers or determiners, both syntactically and semantically. See Ionin and Matushansky (2006) and Zweig (2006) for discussion.

29 Following common use, we write $|x|$ (for x of type e) to mean the number of atoms in x, $|\{z \leq x : ATOM(z)\}|$.

Distributional evidence supporting the adjectival analysis comes both from post-copular predicative positions, as in (19a), and from post-determiner attributive positions, as in (19b), both of which show QNWs other than *most* patterning with regular adjectives, such as *tired*, and differently from proper quantificational determiners, such as *some* and *every* (as well as *most*):

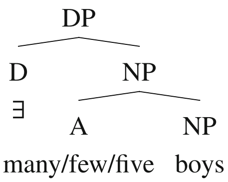
(19) a. They were tired/many/few/five/*some/*all/*every/*most when they left
b. The tired/many/few/five/*some/*all/*every/*most students who came were disappointed

See Landman (2004) for an extended defense of the adjectival analysis. In many cases, of course, QNWs participate in quantificational relations:

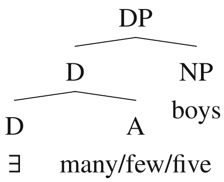
(20) Many/few/five boys smoke

In such cases, adjectival accounts must introduce quantification covertly into the structure. In Landman (2004)'s account, this is done by a type-shifting operation. A different implementation of the same idea would be the introduction of a silent existential quantifier into the determiner position. This silent quantifier can combine with the rest of the noun phrase, as in (21a), or it can combine with the QNW directly, as in (21b)[30]:

(21) (a)

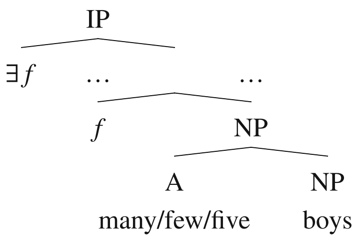


(b)



A different incorporation of quantification into the adjectival approach uses choice functions, a view argued for by Reinhart (1997), Winter (1997), and Kratzer (1998), among others. On this view, the QNW combines with the rest of the noun phrase, after which a choice function, existentially bound from a higher position, selects an appropriate element of the denotation:

(22)



In (22), f is a choice function from sets of individuals to individuals. A sentence such as *Five boys smoke* can be paraphrased, on this view, as saying that there is a way

[30] For (21a), ∃ can be thought of as a silent version of *some*, and its entry can be identical to the one in (14b) above. For (21b), this entry would need to be modified so as to take two restrictors rather than one: first the relevant QNW and then the NP.

of choosing from the set of plural individuals with five atoms, all of them students, such that the predicate *smokes* distributes over this choice.

The adjectival approach has to treat *most*—the standard example showing that natural language quantification goes beyond first order logic—as different from other QNWs. As (19) showed, *most* patterns with quantificational determiners like *every* and *some* in terms of its distribution rather than with QNWs like *many* and *five*. This suggests that, if (17) is correct as stated, *most* must be structurally complex. One natural approach is to treat *most* as the result of combining an adjectival *many* and the superlative *-est*, a decomposition going back to Bresnan (1973):

(23)

D

A | Aff

many | *-est*

Recent work by Hackl (2009) provides both semantics-internal evidence and psycholinguistic evidence in favor of a decomposition of *most* along the lines of (23). Here we will briefly summarize the semantics-internal evidence. A much studied fact of superlative adjectives such as *highest*, *biggest*, *slowest*, etc. is that they give rise to two distinct readings:

(24) John climbed the highest mountain
   a. Absolute: John climbed Mt. Everest
   b. Relative: John climbed a mountain that was higher than any mountain that someone else climbed

Heim (1985, 1999) and Szabolcsi (1986) analyze this property of superlatives in terms of scope. Specifically, they argue that the superlative morpheme *-est* takes DP-internal scope for the absolute reading but DP-external scope for the relative reading.[31]

Like *highest*, *most* can also give rise to two distinct readings, though the forms are slightly different:

(25) a. Proportional: John climbed most of the mountains
   b. Relative: John climbed the most mountains

Hackl observes that the distribution of the readings for *most* is closely related to the distribution of the readings for *highest*: environments that license the relative reading of *highest* also license the relative reading of *(the) most*; environments that allow only the absolute reading of *highest* allow only the proportional reading of *most*.[32] This seems to call for a unified account of superlatives and *most*, and Hackl provides such an account: he develops a semantics for *-est* that derives the proportional reading of

[31] See Farkas and Kiss (2000), Sharvit and Stateva (2002), and Krasikova (2012) for proposals in which the relative reading is derived without a DP-external attachment of *-est*.

[32] Hackl provides German data that show the same pattern of the different readings and their pattern of availability. The German data are perhaps even more striking since, differently from the English forms in (25), the proportional and the relative uses of *most* have the exact same form.

most as a special case of the absolute reading of superlatives, while the relative reading is derived via high scope of *-est*, along the lines of Heim and Szabolcsi.

The decomposition and the semantics he proposes for *-est* allow Hackl to address an additional puzzle. While *most* has the two readings above, *fewest* has only the relative reading:

(26) a. * Proportional: John climbed fewest of the mountains
b. Relative: John climbed the fewest mountains

Hackl shows that, on his decompositional account, the proportional reading of *few-est* gives rise to triviality, which can be used to account for the systematic absence of complex quantificational determiners of the form *few+-est* with the meaning 'less than half'. If we assume further, as we did in (17), that denotations such as 'more than half' or 'less than half' cannot be assigned to simplex quantificational determiners and can only be derived via composition, we predict that any quantificational determiner with the meaning 'more than half' will be complex and that no quantificational determiner, whether simplex or with superlative morphology, can mean 'less than half'.[33]

Gajewski (2010) provides an additional argument for the decomposition of *most*, from the domain of NPI licensing.[34] Building on Hackl, Gajewski shows that the decomposition of *most* into *many* and *-est*, along with the appropriate presuppositional analysis of *-est* and von Fintel (1999)'s notion of *Strawson entailment*, provides a handle on the puzzling ability of *most* to license NPIs within its restrictor, as in *Most students with any brains have already left town*, despite the fact that the restrictor of *most* is clearly not downward entailing.

Coming back to the status of the constraint on lexicalization in (17), recall that we were interested in the analysis of QNWs since these were the main candidates for elements of a category in $\mathcal{H}$—in this case quantificational determiners—that are not generalized conjunction or disjunction. On the adjectival approach, QNWs are always adjectival at core. Adjectives are not elements of $\mathcal{H}$, so simplex lexicalizations of *many*, *few*, and *five* do not threaten (17). In the case of *most*, the decomposition argued for by Hackl (2009), if correct, means that this quantificational determiner is complex, again allowing us to keep (17) without change.

Contrasting fundamentally with the adjectival approach is the view, going back to Montague (1974), Barwise and Cooper (1981), and Partee (1987), according to which QNWs are always quantificational at core. On this quantificational view, *many boys smoke*, *few boys smoke*, *five boys smoke*, and *every boy smokes* all have the same general structure: $Q(boy)(smoke)$. In each case, Q can have a denotation such as:

(27) a. $[\![\mathrm{Q}_{every}]\!] = \lambda f_{et} \lambda g_{et}.\{x : f(x)\} \subseteq \{x : g(x)\}$
b. $[\![\mathrm{Q}_{five}]\!] = \lambda f_{et} \lambda g_{et}.|\{x : f(x)\} \cap \{x : g(x)\}| \geq 5$
c. $[\![\mathrm{Q}_{few}]\!] = \lambda f_{et} \lambda g_{et}.|\{x : f(x)\} \cap \{x : g(x)\}| \leq d$, for some contextually determined d

[33] See Hunter et al. (2009), however, for evidence suggesting that children can learn items with the meaning 'less than half'.

[34] For other developments building on Hackl's analysis of *most* see Kotek et al. (2011), Solt (2011), and Szabolcsi (2012).

d. $[\![Q_{many}]\!] = \lambda f_{et} \lambda g_{et}.|\{x : f(x)\} \cap \{x : g(x)\}| \geq d$, for some contextually determined d

If the denotations for the QNWs in (27) are indeed given directly to simplex lexical items, we would have to revise (17). As Szabolcsi (2010) notes, however, the quantificational view has generally remained silent on the internal structure of Q.[35] It is in fact quite compatible with a decompositional view of QNWs along the lines of (21b) above.[36] From the perspective of the present proposal, (17) is compatible with the quantificational view, though it would commit it to a decompositional approach to all QNWs.[37,38]

3 An ordering-based perspective

We now have a working version of Horn's account. (17) restricts the space of operators that can be lexicalized in principle to the corners of the squares in the relevant categories, after which Horn's two principles apply to block the lexicalization of the O corners. The price we have paid came mostly in the form of the stipulated entries for the A and I corners in the domain of connectives. In this section we would like to briefly consider the question of whether these stipulations can be made less arbitrary.

We believe that a helpful perspective is the ordering-based one, proposed for natural language by Keenan and Faltz (1978, 1985) and pursued fruitfully in various areas since.[39,40] From an ordering-based perspective, conjunction and disjunction are not just any truth tables. They correspond to minimization and maximization, respectively, on the assumption that the domain of truth values is ordered and that $0 < 1$ (where 0 stands for false, and 1 stands for true). The truth value of a conjunctive sentence such as *John is tall, and Mary is short* is the minimum truth value of the conjuncts. If both conjuncts are true ($= 1$), this minimum is also 1, and the whole sentence is

[35] A notable exception, of course, is Keenan and Stavi (1986).

[36] For the case of *most*, if Hackl (2009) is right, the quantificational approach, like the adjectival one, is in fact obliged to use such a decomposition.

[37] If the present form of (17) is to be maintained alongside the quantificational view, a question that should be answered—but that we will not investigate here—is whether there is independent evidence for this decomposition across QNWs.

[38] Questions similar to those concerning QNWs arise with respect to the indefinite article. Russell (1919) analyzed the indefinite article as an existential quantifier, and a long philosophical tradition has defended this view in the face of various well-known challenges. The influential analyses of Kamp (1981) and Heim (1982) have proposed treating indefinite noun phrases as non-quantificational, with possible interactions with quantificational elements elsewhere in the structure, such as existential closure over individuals or choice functions. See Heim (1990) and Ludlow and Neale (1991) for a defense of the Russellian analysis. Note that in the case of the indefinite article, a simplex quantificational treatment as a variant of *some* is equally compatible with (17).

[39] See in particular the ordering-based accounts of the nominal domain in work by Link (1983), Landman (1989a,b), and others, and the semantic accounts of weak islands in work by Szabolcsi and Zwarts (1993), Rullmann (1995), Fox and Hackl (2006), Abrusán (2011), and others. See Szabolcsi (2010) and Szabolcsi et al. (2012) for discussion of the ordering-based perspective in view of morpho-semantic evidence in different languages.

[40] See Gazdar and Pullum (1976) and Gazdar (1979) for a different perspective that makes the choice of conjunction and disjunction as primitives natural.

true. Otherwise, there is at least one conjunct denoting 0, so the minimum is also 0, and the whole sentence is false.[41] To move from sentential conjunction to sentential disjunction, on the ordering-based view, replace 'minimum' with 'maximum' (or keep 'maximum' and reverse the ordering of the truth values). For example, the truth value of *John is tall, Mary is short, or Kim is sad* is the maximum of the truth values of the disjuncts. If at least one is 1, the maximum is also 1, and the whole disjunction is true. Only if all disjuncts are 0 is the maximum 0 and the sentence false.

Since we take D_t to be both finite and totally ordered, we can define sentential *and* and *or* as minimum and maximum. In general, though, we often have a higher type τ that ends in t and has an ordering that is induced by the ordering on D_t:

(28) Let τ be a type that ends in t. $\leq_\tau$ is defined as:

a. $\leq_t := \{<0, 0>, <0, 1>, <1, 1>\}$ if $\tau = t$, and

b. $\{<x, y> \in D_\tau \times D_\tau : \forall z_{\sigma_1}.x(z) \leq_{\sigma_2} y(z)\}$ if $\tau = \sigma_1\sigma_2$

In such cases, what we are interested in is not a minimal or maximal element but the more general *infimum* (= greatest lower bound) and *supremum* (= least upper bound). We assume the following standard definitions (see, for example, Davey and Priestley 2002):

(29) Let $<P, \leq>$ be an ordered set and let $S \subseteq P$.

a. An element $x \in P$ is an *upper bound* of S if $s \leq x$ for all $s \in S$. An element $x \in P$ is a *lower bound* of S if $x \leq s$ for all $s \in S$.

b. If the set of upper bounds of S has a least element x, we will say that x is the *supremum* (or *least upper bound*) of S, written sup S. If the set of lower bounds of S has a greatest element x, we will say that x is the *infimum* (or *greatest lower bound*) of S, written inf S.

Using (29), the conjunctive operators translate into inf and the disjunctive ones into sup. The ordering-based translations of the operators discussed in Sect. 2.1 above are as follows:

(30) Coordination in an ordering-based framework. Let τ be a type that ends in t. Then:

a. $[\![\text{and}]\!] = \lambda P \in {D_\tau}^n.\inf\{P_i : i = 1 \ldots n\}$

b. $[\![\text{or}]\!] = \lambda P \in {D_\tau}^n.\sup\{P_i : i = 1 \ldots n\}$

(31) Basic quantificational determiners in an ordering-based framework:

a. $[\![\text{every}]\!] = \lambda f_{et}.\inf\{\lambda P_{et}.P(x) : f(x)\}$

b. $[\![\text{some}]\!] = \lambda f_{et}.\sup\{\lambda P_{et}.P(x) : f(x)\}$

[41] There is a common way to present the ordering-based view that does involve lookup in truth tables. This alternative presentation starts from *and* and *or*, where these operators are given by their standard truth tables, and then proceeds to define an ordering relation derivatively. Despite the reference to truth tables, however, this alternative presentation does not treat the truth tables for its basic operators as arbitrary: they are subject to the requirement that the operators given by them impose an appropriate structure (usually a boolean algebra) on their domain.

(32) Modal verbs in an ordering-based framework (where R is an appropriately selected accessibility relation):

a. $[\![\text{must}]\!]^w = \lambda p_{st}.\inf\{p(w') : wRw'\}$

b. $[\![\text{may}]\!]^w = \lambda p_{st}.\sup\{p(w') : wRw'\}$

To implement the ordering-based perspective, we keep the constraint on lexicalization in (17) but provide different meanings for $\sqcap$ and $\sqcup$: while earlier $\sqcap$ denoted the truth table for conjunction, it now denotes inf; and while earlier $\sqcup$ denoted the truth table for disjunction, it now denotes sup.[42]

In terms of their outcomes for the cases we have seen, the ordering-based approach and the earlier truth-tabular approach are identical. The difference is one of perspective: the truth-tabular approach states that the simplex operators across $\mathcal{H}$ are defined in terms of two seemingly arbitrary truth tables (out of sixteen potential denotations of this type), the one for conjunction and the one for disjunction; the ordering-based approach, on the other hand, states that the simplex operators are defined in terms of what are arguably the simplest ordering-based operators, inf and sup. If the domains that end in t are indeed ordered, this change of perspective would be warranted. Ultimately, of course, one would want to find empirical arguments that bear on the matter.

4 Markedness, contradiction, and the choice of alternatives

4.1 Logically independent alternatives

Having discussed the building blocks needed to situate Horn's proposal within its broader typological context and to capture the requisite markedness distinctions to make it work, we wish to turn to the proposal itself and to a concern about the notion of implicature used in the Gricean condition. The concern is the following: the definition of OSIs in (5) was intentionally simplistic, as mentioned in Sect. 1.2; this simplistic definition was convenient for purposes of presentation, but it is at odds with a significant body of literature on scalar implicatures, and it would clearly be better if the notion of OSIs used to account for the lexicalization asymmetry were the same as the one used for scalar implicatures elsewhere. The present section attempts to address this concern. We will start by discussing a modification of the notion of OSIs used so far that allows OSIs to be based on the negation of alternatives that are logically independent of the assertion. This modification will bring the definition of OSIs closer to various accounts of scalar implicatures in the literature, but we will see that it introduces additional hypothetical inventories, which will prevent Horn's negation condition from choosing a single winner. We will then discuss a way in which work on the roles of contradiction and markedness in the computation of implicatures can help us escape the problem for the modified account.

The Gricean condition works with a naive notion of scalar implicature, as defined in (5) above, repeated here:

[42] As a reviewer points out, inf and sup are duals, a relationship similar to the one holding between conjunction and disjunction.

(33) Operator-level Scalar Implicature (repeated from (5)): Let Y be a set of operators. For any two operators y and z we say that z is an *operator-level scalar implicature* (OSI) of y given Y, written $y \rightsquigarrow_Y z$, *iff* both of the following conditions hold:

a. $\neg z \in Y$

b. $\neg z \sqsubset y$

If x is an operator, (33) licenses OSIs that are negations of alternatives that are *strictly stronger* than x. However, there are various observations in the literature on scalar implicatures showing inferences that seem to rely on the negation of alternatives that are not stronger than the assertion but instead are independent of it. To take an early example by Horn (1972), there is no entailment relation between committing a felony and committing a misdemeanor; and yet, *John has committed a misdemeanor* implies that it is not the case that John has committed a felony. Building in part on Horn's observations, Hirschberg (1985/1991) provides many additional examples of scalar implicatures that are constructed on-the-fly and involve the negation of alternatives that are logically independent of the assertion and are negated only due to the particular circumstances of the utterance. Simplifying one of Hirschberg's better-known examples, *John typed the letter* and *John mailed the letter* do not stand in a logical entailment relation, but in an appropriate context, uttering the former can suggest that the latter is false. More recent accounts of implicature that rely on the negation of alternatives that do not stand in an entailment relation with the assertion include van Rooij and Schulz (2004), Sevi (2005), Spector (2006), Fox (2007), and Chemla (2009). All things being equal, we would prefer a notion of implicature for operators that is in line with the notion of implicature used for propositions elsewhere.

Definition (34) revises (33) by weakening the condition on alternatives: instead of negating only alternatives that are strictly stronger than y, we now negate all alternatives that are non-weaker than y.

(34) Operator-level Scalar Implicature (second attempt; to be revised in (36)): Let Y be a set of operators. For any two operators y and z we say that z is an *operator-level scalar implicature* (OSI) of y given Y, written $y \rightsquigarrow_Y z$, *iff* both of the following conditions hold:

a. $\neg z \in Y$

b. $y \not\sqsubseteq \neg z$

(34) licenses all the inferences that were valid before (and adds to them inferences based on logically independent alternatives). In particular, the inferences of the form $I \rightsquigarrow \neg A (= O)$ will still be valid, deriving the coverage $[\{A, I, E\}] = [\{A, I, E, O\}]$. And inferences of the form $O \rightsquigarrow \neg E (= I)$ will still be valid, deriving the coverage $[\{A, E, O\}] = [\{A, I, E, O\}]$. The modified notion of implicature in (34), then, still allows the Gricean condition to derive the two inventories in (35a) and (35b). And at this point, one might hope that the negation condition (9) would rule out (35b) as before.

This, however, is not to be. By using (34), we now derive not only the two inventories in (35a) and (35b) but also two additional inventories, (35c) and (35d).[43] (35c) is brought in since we now have $[\{I, E, O\}] = [\{A, I, E, O\}]$ due to $I \rightsquigarrow \neg O(= A)$. (35d) is brought in since we now have $[\{A, I, O\}] = [\{A, I, E, O\}]$ due to $O \rightsquigarrow \neg I(= E)$. Since the unattested inventory (35d) ($= \{A, I, O\}$) has as many instances of negation as the attested (35a) ($= \{A, I, E\}$), the negation condition can no longer do its job.

(35) a. $\{A, I, E\}$ (where $I \rightsquigarrow \neg A(= O)$)
b. $\{A, E, O\}$ (where $O \rightsquigarrow \neg E(= I)$)
c. $\{I, E, O\}$ (where $I \rightsquigarrow \neg O(= A)$)
d. $\{A, I, O\}$ (where $O \rightsquigarrow \neg I(= E)$)

4.2 Detour: contradiction and innocent exclusion

Is the idea of basing our account on implicatures that can negate logically independent alternatives doomed? We think not. In fact, we believe that by paying closer attention to the notion of implicature that is used we can not only maintain the negation of logically independent alternatives but also simplify the account in the process by eliminating the negation condition altogether. The key is to take into account the role of contradiction in computations that involve negating alternatives, such as scalar implicatures, answers to questions, and the semantics of the focus-sensitive operator *only*. This role has been explored in various works in the literature. Here we follow Fox (2007), who builds in turn on work by Groenendijk and Stokhof (1984) and Sauerland (2004). This discussion is largely independent of the question of whether only stronger alternatives are negated.

Suppose the speaker says ϕ = *Bill or Bob came to the party*. The disjunctive ϕ is asymmetrically entailed by ψ = *Bill came to the party*. If we could simply negate all alternatives that are better (strictly stronger, or maybe also logically independent), we could hope to use ϕ to imply that Bill did not come; but of course, it implies no such thing. The reason, in this case, seems quite intuitive: ϕ should deny that Bill came, but by the same reasoning it would also indicate that the stronger ψ' = *Bob came to the party* is false, thus denying that Bob came. But if we draw both inferences (that is, both $\neg\psi$ and $\neg\psi'$), we conclude that Bill did not come and that Bob did not come, which contradicts the assertion.

The literature on implicatures has devised a variety of solutions to avoid such contradictory inferences. On the approach developed by Fox (2007), computations that involve the negation of alternatives are subject to a requirement, named *innocent exclusion* and defined as in (37), that explicitly prevents contradictory inferences from arising. In addition to avoiding contradictory inferences, innocent exclusion also ensures that contradiction will not be avoided by making arbitrary choices. In the example above, one could imagine a computation that negates ψ and ignores ψ', thus

[43] In each of the four inventories in (35), the OSI $\rightsquigarrow$ is taken to be computed relative to the inventory under consideration.

avoiding the problematic contradiction. This move, however, would lead us to the conclusion that Bob did come to the party, which is an arbitrary conclusion. Innocent exclusion avoids such moves by ensuring that the only alternatives that are negated are those that do not give rise to arbitrary conclusions. In the example above, we can conclude from ϕ that ϕ' = *Bill and Bob came to the party* is false, since this does not force us to make any other alternative true; but we are prevented from negating either of ψ or ψ'.

(36) OPERATOR-LEVEL SCALAR IMPLICATURE (third attempt; to be revised in (40)): Let Y be a set of operators. For any two operators y and z we say that z is an *operator-level scalar implicature* (OSI) of y given Y, written $y \rightsquigarrow_Y z$, *iff* both of the following conditions hold:

a. $\neg z \in Y$

b. $\neg z$ is *innocently excludable* given y and Y

(37) An element x is *innocently excludable* given an element a and a set A if x is in every maximal subset of A that can be negated consistently with a, $x \in IE(a, A)$

a. $IE(a, A) := \bigcap\{B \subseteq A : B$ is a maximal set in A s.t.$\neg B \cup \{a\}$ is consistent$\}$

b. $\neg B := \{\neg b : b \in B\}$

Let us summarize this brief detour. Both the original definition of OSI using strictly stronger alternatives in (5) and the modification that allows the negation of logically independent alternatives in (34) run into the problem of contradictory inferences. When we concluded that (34) yielded unwanted inventories, then, we did this on the basis of a problematic definition of OSIs. Before giving up on the idea of negating logically independent alternatives, it would therefore make sense to see whether a more appropriate version of the definition, one in which contradictions are avoided, still yields the unwanted inventories. The definition in (36) attempted to improve on the definition in (34) by incorporating the notion of innocent exclusion. In the next subsection we will see that this allows us to avoid the problem of unwanted inventories.

4.3 Eliminating inventories using contradiction

Let us return now to the derivation of inventories, and let us focus first on the two putative inventories (35b) $= \{A, E, O\}$ and (35d) $= \{A, I, O\}$ that rely on inferences arising from the use of the O corner. Furthermore, as stated in our constraint on lexicalization, we are assuming that all four corners of the square are lexicalizable in principle, A and I as simplex, and E and O as a complex of a positive corner and negation. So we are supposing that we have the full hypothetical inventory $\{A, I, E, O\}$ lexicalized in some domain and that an utterance of O is made. The notion of implicature in (34), which allowed logically independent alternatives to be negated but did not incorporate contradiction avoidance, licenses two inferences from this utterance of O: $O \rightsquigarrow \neg E (= I)$ and $O \rightsquigarrow \neg I (= E)$ (whereas only the former was licensed by the earlier definition in (33)). These two OSIs licensed two unwanted inventories

(35b) and (35d). Note, however, that these two inferences are mutually contradictory. The contradiction-free definition in (36), then, will not derive either of the problematic OSIs. This, in turn, means that instead of obtaining two O-based inventories, (35b) and (35d), we will obtain neither. This is encouraging: we are maintaining the use of the notion of implicature that negates logically independent alternatives, used elsewhere in the literature, and we have just eliminated two unwanted inventories by using a notion of contradiction avoidance that has been argued for independently, rather than by invoking a special-purpose mechanism such as the negation condition.[44]

Things seem less promising when we turn to an utterance of I, again considered in the context of lexicalizing the entire square in some particular domain. The definition of OSI in (34), which allowed logically independent alternatives to be negated but did not incorporate contradiction avoidance, again licenses two inferences from this utterance (this time of I): $I \rightsquigarrow \neg A (= O)$ and $I \rightsquigarrow \neg O (= A)$. And again, the two inferences are mutually contradictory. The contradiction-free definition in (36), then, will not derive either of the OSIs. And again we will be left without either of the two relevant inventories, in this case (35a) $= \{A, I, E\}$ and (35c) $= \{I, E, O\}$. In this case, however, only one of the inventories—the one in (35c)—was problematic. The other inventory—the one in (35a)—is the attested one, and we certainly would not like to lose it.

Let us summarize our last few steps. We have tried to modify the definition of OSIs so as to bring it closer to accounts of implicature. However, adopting the negation of logically independent alternatives in (34) added more unattested inventories than can be handled by the system, while incorporating the mechanism of innocent exclusion in (36) to avoid contradiction eliminated all the inventories, including the attested one.

While the elimination of inventories effected by the last modification is clearly too sweeping, note that a minimal modification would have given us exactly the correct result: if we could somehow remove the potential inference $I \rightsquigarrow \neg O (= A)$ while keeping everything else without change, the contradiction avoidance mechanism would still prevent the two contradictory O-based inferences $O \rightsquigarrow \neg E (= I)$ and $O \rightsquigarrow \neg I (= E)$ from arising, while the good I-based inference $I \rightsquigarrow \neg A (= O)$ would now remain without a contradictory inference. This, in turn, means that of all the potential inferences, only the good I-based one would remain, and we would be left only with the attested I-based inventory in (35a). In the next subsection we will attempt to bring about this state of affairs.

4.4 The role of markedness

Our discussion so far has assumed that within an inventory, every element can serve as an alternative to every other element. Starting with Horn (1972), however, theories of implicature have been careful to restrict the possible alternatives that can enter into the computation of scalar implicature. We could rely on such a formal restriction in order to eliminate the problematic OSI $I \rightsquigarrow \neg O (= A)$. As a first step, let us add the

[44] Crucially, it must be impossible for other factors to eliminate one of these formal alternatives while keeping the other. See Fox and Katzir (2011) for an argument that this is indeed the case.

following stipulation about the sets of formal alternatives associated with I and O, which we write as $Alt(I)$ and $Alt(O)$:

(38) Stipulation regarding alternatives:

a. $A \in Alt(I)$, $O \notin Alt(I)$

b. $I \in Alt(O)$, $E \in Alt(O)$

(38) states that O has both E and I as alternatives (as assumed above), while I has A but crucially not O as an alternative. We have not changed anything about our assumptions regarding O. In particular, the contradiction-avoiding mechanism in (36) will still ensure that an utterance of O will lead to neither of the contradictory inferences $\neg E$ and $\neg I$, as just explained, which in turn will ensure that we obtain neither of the unattested inventories in (35d) and (35b). For I, on the other hand, things are now quite different. Since O is no longer an alternative, the potential inference $I \rightsquigarrow \neg O(= A)$ is no longer relevant, and an utterance of I will now have only one potential inference to consider, namely $I \rightsquigarrow \neg A(= O)$. This means that contradiction is no longer a threat, and an utterance of I will indeed have the OSI $I \rightsquigarrow \neg A(= O)$, which in turn will license the attested inventory (35a) through the Gricean condition (3). In other words, stipulating (38) allows the contradiction-avoiding definition of OSI in (36) to generate one single OSI from within a square in a particular domain, namely $I \rightsquigarrow \neg A(= O)$, which in turn means that only one three-corner inventory in that domain will cover the full square, namely the attested (35a). Note that, since there is now only one three-corner inventory that survives the Gricean condition, the tie-breaking negation condition in (9) has become superfluous. In other words, by adopting two modifications that have been argued to be needed independently of our present discussion—the negation of logically independent alternatives and the avoidance of contradiction—and by adopting the stipulation in (38), we can simplify Horn's account by removing the negation condition. The price, at this point, seems to be the stipulation of alternatives. Does this characterization of alternatives follow from any general consideration?

We believe that markedness provides the necessary general consideration. Let us look more closely at the alternatives that we stipulated in (38). For O, which we analyzed above as $\neg A$, the alternatives are E, analyzed as $\neg I$, and I. For I, the alternatives are A but not O, analyzed as $\neg A$. For O, in other words, we have one alternative that is equally marked and one that is strictly less marked. For I, we have one alternative only, which is equally marked, and in particular we do not have a strictly more marked alternative. If, as has sometimes been suggested, the alternatives to ϕ are at most as marked as ϕ (see Horn 1984, 2000; Katzir 2007, and references therein), we can derive (38) rather than stipulating it. Note that we obtain the result by making markedness part of the definition of the alternatives used in the Gricean condition rather than a separate tie-breaking constraint as in Horn's account.[45]

[45] Above we have analyzed the markedness of the negative corners as involving structural complexity. If correct, this allows us to make the idea of markedness-sensitive alternatives more concrete by stating it within a general theory of alternatives that is sensitive to structural complexity, such as the one developed in Katzir (2007) and Fox and Katzir (2011) based on evidence that is independent of the current discussion.

4.5 Final state of the system

We can now present the final state of the system. The space of potentially lexicalizable operators is restricted according to (39), which limits lexicalization within categories in $\mathcal{H}$ to the simplex positive corners and the marked negative corners, each composed of negation and a positive corner. Within this space, the notion of implicature, in (40), allows the negation of logically independent alternatives, avoids contradiction, and relies on markedness to define alternatives. The Gricean condition, repeated in (41), is the only condition we have (the negation condition is no longer needed).

(39) Constraint on lexicalization (negation-aware): For every category $C \in \mathcal{H}$, both of the following hold.

a. Basic case: The only simplex operators in C are A (the rule-based extension of $\bigsqcap$ in C) and I (the rule-based extension of $\bigsqcup$ in C)

b. Marked case: For an operator μ defined as in (39a), it may be possible to lexicalize $\neg\mu$, and the result is marked

(40) Operator-level Scalar Implicature (final version): Let Y be a set of operators. For any two operators y and z we say that z is an *operator-level scalar implicature* (OSI) of y given Y, written $y \rightsquigarrow_Y z$, *iff* all three of the following conditions hold:

a. $\neg z \in Y$

b. $\neg z$ is *innocently excludable* given y and $Alt(y, Y)$

c. $Alt(y, Y)$ is the set of elements in Y that are at most as marked as y

(41) Gricean Condition: Let X and Y be two inventories of logical operators such that $[X] = [Y]$. If $Y \subset X$, X cannot be lexicalized.

5 Discussion

We looked at a typological puzzle, due to Horn (1972), regarding the lexicalization of logical operators: in instantiations of the square of opposition across categories and languages, the O corner is systematically absent. We discussed Horn's pragmatic solution, in which the O corner is blocked due to a scalar implicature from the I corner, along with a separate condition that compares inventories and selects the least marked among them.

We noted that for pragmatic accounts such as Horn's to work, the logical operators that can potentially be lexicalized as simplex must be limited to the positive corners of squares of opposition. We followed Keenan and Stavi (1986) in restricting the elements within categories in $\mathcal{H}$ that can be lexicalized as simplex to a small set of primitives. With this restriction, operators that are not on a square of opposition cannot be lexicalized in the first place, the markedness of negation is expressed in structural terms, and the O corner can be eliminated pragmatically.

We then looked for a way to make the stipulations more natural. Building on Keenan and Faltz (1985), we suggested that the domain-general definitions are based on ordering, and that the only primitive operators are $\inf(= A)$ and $\sup(= I)$.

Finally, we used work on the role of non-stronger alternatives in the computation of scalar implicatures to suggest a modification of the proposal. We addressed an apparent challenge to the modification by ensuring that the computation of implicatures is contradiction free, and we addressed an additional challenge by incorporating markedness directly into the definition of the alternatives. The resulting system maintained Horn's Gricean condition but dispensed with the negation condition, thus simplifying the original account.

Acknowledgments We thank Ash Asudeh, Johan van Benthem, Danny Fox, Kai von Fintel, Tova Friedman, Irene Heim, Larry Horn, Fred Landman, Philippe Schlenker, Anna Szabolcsi, and Ida Toivonen; the audiences at Bar Ilan University, Carleton University, Cornell University, Hebrew University, MIT, NYU, and Tel Aviv University; and three anonymous reviewers for *L&P* as well as Paul Portner. RK has been supported by ISF grant 187/11.

References

Abels, K., & Martí, L. (2010). A unified approach to split scope. *Natural Language Semantics, 18*, 435–470.

Abrusán, M. (2011). Presuppositional and negative islands: A semantic account. *Natural Language Semantics, 19*, 257–321.

Barwise, J., & Cooper, R. (1981). Generalized quantifiers and natural language. *Linguistics and Philosophy, 4*, 159–219.

Béziau, J.-Y. (2003). New light on the square of oppositions and its nameless corner. *Logical Investigations, 10*, 218–233.

Blanché, R. (1953). Sur l'opposition des concepts. *Theoria, 19*, 89–130.

Blanché, R. (1969). *Structures intellectuelles*. Paris: J. Vrin.

Bresnan, J. (1973). Syntax of the comparative clause construction in English. *Linguistic Inquiry, 4*, 275–343.

Chemla, E. (2009). Similarity: Towards a unified account of scalar implicatures, free choice permission and presupposition projection. Under revision for Semantics and Pragmatics.

Davey, B. A., & Priestley, H. A. (2002). *Introduction to lattices and order* (2nd ed.). Cambridge: Cambridge University Press.

de Swart, H. (2000). Scope ambiguities with negative quantifiers. In K. von Heusinger & U. Egli (Eds.), *Reference and anaphoric relations* (pp. 109–132). Dordrecht: Kluwer.

Farkas, D. F., & Kiss, K. E. (2000). On the comparative and absolute readings of superlatives. *Natural Language and Linguistic Theory, 18*, 417–455.

Fox, D. (2007). Free choice disjunction and the theory of scalar implicatures. In U. Sauerland & P. Stateva (Eds.), *Presupposition and implicature in compositional semantics* (pp. 71–120). New York: Palgrave-Macmillan.

Fox, D., & Hackl, M. (2006). The universal density of measurement. *Linguistics and Philosophy, 29*, 537–586.

Fox, D., & Katzir, R. (2011). On the characterization of alternatives. *Natural Language Semantics, 19*, 87–107.

Gajewski, J. (2010). Superlatives, NPIs and *most*. *Journal of Semantics, 27*, 125–137.

Gazdar, G. (1979). *Pragmatics: Implicature, presupposition and logical form*. New York: Academic Press.

Gazdar, G. (1980). A cross-categorial semantics for coordination. *Linguistics and Philosophy, 3*, 407–409.

Gazdar, G., & Pullum G. K. (1976). Truth functional connectives in natural language. In *Papers from the regional meeting of the Chicago Linguistic Society* (Vol. 12, pp. 220–234), Chicago, IL.

Geurts, B. (1996). On no. *Journal of Semantics, 13*, 67–86.

Grice, P. (1989). *Studies in the way of words*. Cambridge: Harvard University Press.

Groenendijk, J., & Stokhof, M. (1984). *Studies in the semantics of questions and the pragmatics of answers*. Doctoral Dissertation, Universiteit van Amsterdam, Amsterdam.

Hackl, M. (2009). On the grammar and processing of proportional quantifiers: Most versus more than half. *Natural Language Semantics, 17*, 63–98.

Heim, I. (1982). *The semantics of definite and indefinite noun phrases*. Doctoral Dissertation, University of Massachusetts, Amherst.

Heim, I. (1985). Notes on comparatives and related matters. Ms., University of Texas at Austin.

Heim, I. (1990). E-type pronouns and donkey anaphora. *Linguistics and Philosophy, 13*, 137–178.

Heim, I. (1999). Notes on superlatives. MIT lecture notes. Ms., MIT.

Higginbotham, J., & May, R. (1981). Questions, quantifiers and crossing. *The Linguistic Review, 1*, 1–41.

Hirschberg, J. (1985/1991). *A theory of scalar implicature*. New York: Garland.

Hoeksema, J. (1999). Blocking effects and polarity sensitivity. In J. Gerbrandy, M. Marx, M. de Rijke, & Y. Venema (Eds.), *Jfak: Essays dedicated to Johan van Benthem on the occasion of his 50th birthday*. Amsterdam: Amsterdam University Press.

Horn, L. (1972). *On the semantic properties of the logical operators in English*. Doctoral Dissertation, UCLA.

Horn, L. (1984). Toward a new taxonomy for pragmatic inference: Q-based and R-based implicatures. In D. Schiffrin (Ed.), *Meaning, form, and use in context* (pp. 11–42). Washington: Georgetown University Press.

Horn, L. (1989). *A natural history of negation*. Chicago: University of Chicago Press.

Horn, L. (1990). Hamburgers and truth: Why Gricean inference is Gricean. In *BLS* (Vol. 16, pp. 454–471).

Horn, L. (2000). From IF to IFF: Conditional perfection as pragmatic strengthening. *Journal of Pragmatics, 32*, 289–326.

Horn, L. (2011). *Histoire d'*O: Lexical pragmatics and the geometry of opposition* (pp. 383–416). Bern: Peter Lang.

Hunter, T., & Lidz, J. (2012). Conservativity and learnability of determiners. *Journal of Semantics, 29*(3).

Hunter, T., Lidz, J., Wellwood, A., & Conroy, A. (2009). Restrictions on the meaning of determiners: Typological generalisations and learnability. In E. Cormany & S. Ito (Eds.), *Proceedings of SALT XIX*. Ithaca, NY: CLC Publications.

Ionin, T., & Matushansky, O. (2006). The composition of complex cardinals. *Journal of Semantics, 23*, 315–360.

Jacobs, J. (1980). Lexical decomposition in Montague grammar. *Theoretical Linguistics, 7*, 121–136.

Jaspers, D. (2005). *Operators in the lexicon: On the negative logic of natural language*. Doctoral Dissertation, University of Leiden, Leiden.

Kamp, H. (1981). A theory of truth and semantic representation. In J. Groenendijk (Ed.), *Formal methods in the study of language*. Amsterdam: Mathematical Center.

Katzir, R. (2007). Structurally-defined alternatives. *Linguistics and Philosophy, 30*, 669–690.

Katzir, R., & Singh R. (2009). On the absence of XOR in natural language. In P. Égré & G. Magri (Ed.), *Presuppositions and implicatures: Proceedings of the MIT-Paris workshop* (pp. 118–129). Cambridge, MA: MITWPL.

Keenan, E. L., & Faltz, L. (1978). *Logical types for natural language*. UCLA Occasional Papers in Linguistics. Los Angeles, CA: UCLA.

Keenan, E. L., & Faltz, L. (1985). *Boolean semantics for natural language*. Dordrecht: Reidel.

Keenan, E. L., & Stavi, J. (1986). A semantic characterization of natural language determiners. *Linguistics and Philosophy, 9*, 253–326.

Kneale, W., & Kneale, M. (1962). *The development of logic*. Oxford: Oxford University Press.

Kotek, H., Sudo, Y., Howard, E., & Hackl, M. (2011). Three readings of most. In *Proceedings of SALT* (Vol. 21, pp. 353–372).

Krasikova, S. (2012). Definiteness in superlatives. In *Logic, language and meaning* (pp. 411–420). Berlin: Springer.

Kratzer, A. (1981). The notional category of modality. In H. Eickmeyer & H. Rieser (Eds.), *Words, worlds, and contexts* (pp. 38–74). Berlin: Walter de Gruyter.

Kratzer, A. (1986). Conditionals. (Reprinted from *Semantics: An international handbook of contemporary research*, by A. von Stechow & D. Wunderlich, Eds., 1991, Berlin: Walter de Gruyter.)

Kratzer, A. (1998). Scope or pseudoscope? Are there wide-scope indefinites? In S. Rothstein (Ed.), *Events and grammar* (pp. 163–196). Dordrecht: Kluwer.

Landman, F. (1989a). Groups, I. *Linguistics and Philosophy, 12*, 559–605.

Landman, F. (1989b). Groups, II. *Linguistics and Philosophy, 12*, 723–744.

Landman, F. (2004). *Indefinites and the type of sets*. Oxford: Blackwell.

Lewis, D. (1975). Adverbs of quantification. In E. Keenan (Ed.), *Formal semantics of natural language*. Cambridge: Cambridge University Press.

Link, G. (1983). The logical analysis of plurals and mass terms: A lattice-theoretic approach. In R. Bäuerle, C. Schwarze, & A. von Stechow (Eds.), *Meaning, use, and interpretation of language* (pp. 303–323). Berlin: De Gruyter.

Löbner, S. (1983). Phase quantification: A uniform treatment of some quantifiers from different categories. In *Proceedings of the second Japanese-Korean Joint workshop on formal grammar at Kyoto*, pp. 127–140, The Logico-Linguistic Society of Japan, Japan.

Ludlow, P., & Neale, S. (1991). Indefinite descriptions: In defense of Russell. *Linguistics and Philosophy, 14*, 171–202.

Matthewson, L. (2001). Quantification and the nature of crosslinguistic variation. *Natural Language Semantics, 9*, 145–189.

Matthewson, L. (2011). Strategies of quantification in st'át'imcets and the rest of the world. Ms., University of British Columbia.

McCawley, J. D. (1972). A program for logic. In *Semantics of natural language* (pp. 157–212). Dordrecht: Reidel.

Montague, R. (1974). The proper treatment of quantification in English. In R. H. Thomason (Ed.), *Formal philosophy: Selected papers of Richard Montague*. New Haven, CT: Yale University Press.

Moretti, A. (2012). Why the logical hexagon? *Logica Universalis* (pp. 1–39). doi:10.1007/s11787-012-0045-x.

Parsons, T. (1997). The traditional square of opposition: A biography. *Acta Analytica, 18*, 23–49.

Partee, B. (1987). Noun phrase interpretation and type shifting principles. In J. Groenendijk, D. de Jongh, & M. Stokhof (Eds.), *Studies in discourse representation theory and the theory of generalized quantifiers*. Dordrecht: Foris.

Partee, B. H., & Rooth, M. (1983). Generalized conjunction and type ambiguity. In R. Bäuerle, C. Schwarze, & A. von Stechow (Eds.), *Meaning, use and interpretation of language* (pp. 362–383). Berlin: de Gruyter.

Reinhart, T. (1997). Quantifier scope: How labor is divided between QR and choice functions. *Linguistics and Philosophy, 20*, 335–397.

Rullmann, H. (1995). *Maximality in the semantics of wh-constructions*. Doctoral Dissertation, University of Massachusetts at Amherst, Amherst, MA.

Russell, B. (1919). *Introduction to mathematical philosophy*. London: Allen and Unwin.

Sauerland, U. (1998). *The meaning of chains*. Doctoral Dissertation, MIT, Cambridge, MA.

Sauerland, U. (2000). No 'no': On the crosslinguistic absence of a determiner 'no'. In *Proceedings of the Tsukuba workshop on determiners and quantification* (pp. 415–444).

Sauerland, U. (2004). Scalar implicatures in complex sentences. *Linguistics and Philosophy, 27*, 367–391.

Seuren, P. (2006). The natural logic of language and cognition. *Pragmatics, 16*, 103–138.

Sevi, A. (2005). *Exhaustivity: A semantic account of 'quantity' implicatures*. Doctoral Dissertation, Tel-Aviv University.

Sharvit, Y., & Stateva, P. (2002). Superlative expressions, context, and focus. *Linguistics and Philosophy, 25*, 453–504.

Solt, S. (2011). How many *Most*'s? In I. Reich, E. Horch, & P. Dennis (Eds.), Proceedings of Sinn und Bedeutung 15 (pp. 565–579). Saarbrücken: Saarland Universiy Press.

Spector, B. (2006). *Aspects de la pragmatique des opérateurs logiques*. Doctoral Dissertation, Université de Paris 7, Paris.

Strawson, P. F. (1952). *Introduction to logical theory*. London: Methuen.

Swanson, E. (2010). Structurally defined alternatives and lexicalizations of XOR. *Linguistics and Philosophy, 33*, 31–36.

Szabolcsi, A. (1986). Comparative superlatives. In N. Fukui, T. Rapoport, & E. Sagey (Eds.), *Papers in theoretical linguistics* (Vol. 8, pp. 245–265). Cambridge, MA: MITWPL.

Szabolcsi, A. (2010). *Quantification*. Cambridge: Cambridge University Press.

Szabolcsi, A. (2012). Compositionality without word boundaries: (The) more and (the) most. In *Proceedings of SALT* (Vol. 22, pp. 1–25).

Szabolcsi, A., Whang, J. D., & Zu, V. (2012). Compositionality questions: Quantifier words and their multi-functional parts. Ms., NYU, June 2012

Szabolcsi, A., & Zwarts, F. (1993). Weak islands and an algebraic semantics for scope taking. *Natural Language Semantics, 1*, 235–284.

van Benthem, J. (1984). Questions about Quantifiers. *The Journal of Symbolic Logic, 49*, 443–466.
van Rooij, R., & Schulz, K. (2004). Exhaustive interpretation of complex sentences. *Journal of Logic, Language and Information, 13*, 491–519.
von Fintel, K. (1999). NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics, 16*, 97–148.
von Fintel, K., & Matthewson, L. (2008). Universals in semantics. *The Linguistic Review, 25*, 139–201.
Winter, Y. (1997). Choice functions and the scopal semantics of indefinites. *Linguistics and Philosophy, 20*, 399–467.
Zeijlstra, H. (2011). On the syntactically complex status of negative indefinites. *Journal of Comparative Germanic Linguistics, 4*, 111–138.
Zweig, E. (2006). Nouns and adjectives in numeral NPs. In L. Bateman & C. Ussery (Eds.), *Proceedings of NELS 35* (pp. 663–675). Amherst, MA: GLSA.