

1st Midterm

Foundations of Mathematics

2020 October 13

M1.1 Let $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}$

- (a) Determine which of the following are elements of A : $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$.
- (b) Determine $|A|$.
- (c) Determine which of the following are subsets of A : $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$

For (d)–(i), determine the indicated sets

- (d) $\emptyset \cap A$
- (e) $\{\emptyset\} \cap A$
- (f) $\{\emptyset, \{\emptyset\}\} \cap A$
- (g) $\emptyset \cup A$
- (h) $\{\emptyset\} \cup A$
- (i) $\{\emptyset, \{\emptyset\}\} \cup A$

M1.2 For the set $S = \{1, 2, \dots, 7\}$, consider the five subsets $A_1 = \{1, 2\}, A_2 = \{3, 4, 5\}, A_3 = \{2, 5, 6\}, A_4 = \{2, 7\}, A_5 = \{1, 2\}$.

- (a) Is the collection $P_1 = \{A_1, A_2, A_3, A_4, A_5\}$ a partition of S ?
- (b) Let $B_i = A_i \setminus A_{i+1}$ for $i = 1, 2, 3, 4$. Is $P_2 = \{B_1, B_2, B_3, B_4\}$ a partition of S ?
- (c) Let $C_i = A_{6-i} \setminus A_{5-i}$ for $i = 1, 2, 3, 4$. Is $P_3 = \{C_1, C_2, C_3, C_4\}$ a partition of S ?

M1.3 Prove or disprove the following:

- (a) If two functions $f : A \rightarrow B$ and $g : B \rightarrow C$ are both bijective, then $g \circ f : A \rightarrow C$ is bijective.
- (b) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. If g is surjective, then $g \circ f : A \rightarrow C$ is surjective.
- (c) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions. If g is injective, then $g \circ f : A \rightarrow C$ is injective.
- (d) There exist functions $f : A \rightarrow B$ and $g : B \rightarrow C$ such that f is not surjective and $g \circ f : A \rightarrow C$ is surjective.
- (e) There exist functions $f : A \rightarrow B$ and $g : B \rightarrow C$ such that f is not injective and $g \circ f : A \rightarrow C$ is injective.

M1.4 Let $A = \{1, 2, 3, 4, 5, 6\}$. The distinct equivalence classes resulting from an equivalence relation R on A are $\{1, 4, 5\}, \{2, 6\}$ and $\{3\}$. What is R ?