2nd Midterm

Foundations of Mathematics Final

2020 Dec 8

F.1 Determine the cardinality of each of the following sets:
(a) $A = \{1, 2, 3, \{1, 2, 3\}, 4, \{4\}\}$
(b) $B = \{x \in \mathbb{R} : |x| = -1\}$
(c) $C = \{m \in \mathbb{N} : 2 < m \leq 5\}$
(d) $D = \{n \in \mathbb{N} : n < 0\}$
(e) $E = \{k \in \mathbb{N} : 1 \leq k^2 \leq 100\}$
(f) $F = \{k \in \mathbb{Z} : 1 \leq k^2 \leq 100\}$.

F.2 Let {S, T} be a partition of the set $\mathbb{N}$ of positive integers and let U be a nonempty subset of $\mathbb{N}$. State the negation of each of the following statements:
(a) Every element of U can be expressed as $x + y$, where $x \in S$ and $y \in T$
(b) For every $x \in S$ and $y \in T$, $xy \in S$
(c) For every element $x \in S$, there is an element $y \in T$ such that $y > x$

F.3 Let A, B and C be nonempty sets and let f, g and h be functions such that $f : A \to B$, $g : B \to C$ and $h : B \to C$. For each of the following, prove or disprove:
(a) If $g \circ f = h \circ f$, then $g = h$
(b) If f is one-to-one and $g \circ f = h \circ f$, then $g = h$

F.4 Prove for every positive integer n that $n^2 + 1$ is not a multiple of 6.